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Near-field diffraction-based focal length determination technique

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ABSTRACT

An accurate and simple technique for determining the focal length of a lens is presented. It consists of measuring the period of the fringes produced by a diffraction grating at the near field when it is illuminated with a beam focused by the unknown lens. In paraxial approximation, the period of the fringes varies linearly with the distance. After some calculations, a simple extrapolation of data is performed to obtain the locations of the principal plane and the focal plane of the lens. Thus, the focal length is obtained as the distance between the two mentioned planes. The accuracy of the method is limited by the collimation degree of the incident beam and by the algorithm used to obtain the period of the fringes. We have checked the technique with two commercial lenses, one convergent and one divergent, with nominal focal lengths $(+100 \pm 1)$ mm and (-100 ± 1) mm respectively. We have experimentally obtained the focal lengths resulting into the interval given by the manufacturer but with an uncertainty of 0.1%, one order of magnitude lesser than the uncertainty given by the manufacturer.

1. Introduction

Accurate characterization of optical systems is crucial for applications and techniques. Among all characteristics of optical systems, the focal length is one of the most important. There are many different techniques for determining the focal length of a lens [1–4]. The simplest one consists of using a very well-collimated laser beam and determining the focal spot position by means of a screen or a two-dimensional camera. Although, this technique is actually an approximation since the focal length is commonly defined from the front principal plane of the lens, and it is usually inaccessible. Other methods based on moiré deflectometry utilize two diffraction gratings and analyze the moiré fringes produced after the second grating when the beam is converging through the lens [5–10]. In Refs. [11,12] two methods with a single diffraction grating are presented. In [11], the method consists of measuring the frequencies at the focal plane of the lens and obtaining the focal length for comparison with the frequencies of the grating. On the other hand, in [12] the method consists of measuring the transverse distances of diffraction maxima in one measurement at the focal plane. In Ref. [13] a Hartmann–Shack wavefront sensor is used to determine the focal plane without knowing the position of the principal plane of the lens. In Ref. [14] one diffraction grating is used to determine the focal length of a lens by measuring the demagnification of the self-images produced by the grating illuminated by a convergent beam. However, only two positions are used and then the accuracy is not optimal. Our impression is that the accuracy of the method can be greatly improved. On the other hand, Tebaldi et al. obtain the focal distance but not the location of the principal plane nor the focal plane.

As a consequence, we propose in this work an improvement of the method proposed in [14] where a very simple and accurate technique for determining the focal length of a lens or lens system is presented. It is based on the self-imaging phenomenon and consists of measuring the period of the converging/diverging self-images produced after the lens when it is illuminated by a collimated beam. On the contrary to ref. [14], our method places the grating before the lens with unknown optical parameters. Thus, it is possible to obtain the position of the principle plane of the lens. It corresponds with the plane in which the period of the self-images equals the period of the grating. Also, at the focal plane the period of the self-images is zero. Then, by extrapolating the values of the period of the self-images at several distances, it is possible to obtain the focal distance. To increase the accuracy of the technique, we use the variogram function for computing the period of the self-images since noise is highly reduced. To check the technique, we measure two lenses, one convergent and one divergent, with nominal focal lengths +100 mm and −100 mm respectively. The uncertainty given by the manufacturer for both lenses is 1% and we obtain an uncertainty of approximately 0.1%, one order of magnitude lesser.

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2. Analytical approach

Let us consider the set-up shown in Fig. 1. It consists of a well-collimated light beam of wavelength λ, described by $U(z) = U_0 e^{ikz}$ that impinges an amplitude Ronchi diffraction grating, DG, with well-known period p. The transmittance of the grating can be expressed as its Fourier expansion series as

$$t(x) = \sum_n a_n e^{iqnx_0}, \tag{1}$$

where x_0 is the transversal coordinate at the grating plane, n are integer numbers, $a_n = \tau sinc\left(n\pi\tau\right)$ are the Fourier coefficients of the grating [15], τ is the fill factor of the grating, and $q = 2\pi/p$. When the grating is illuminated by a plane wave, it produces self-images at the near field that consist of exact replicas of the intensity distribution of the grating [16,17], that appear at multiples of the so-called Talbot distance, $z_T = 2p^2/\lambda$. The propagated field after the grating can be calculated by using the Fresnel approach resulting in

$$U_1(\xi, z) = U_0 \sqrt{\frac{i}{\lambda z}} e^{ikz} \sum_{n=-\infty}^{\infty} a_n e^{2\pi i n \frac{\xi}{p}} e^{i2\pi n^2 z/z_T}, \tag{2}$$

where z is the distance from the grating to the observation plane. After, light propagates to the lens with unknown focal length f'. Considering thin lens approximation, the transmittance of the lens centered at the optical axis in Fresnel regime is given by

$$L(\xi) = e^{-ik\frac{\xi^2}{2f'}}, \tag{3}$$

where $k = 2\pi/\lambda$. We use again Fresnel approach to propagate the optical field from the lens forward resulting in

$$U_2(x, z) = \sqrt{\frac{i}{\lambda z}} e^{ikz} \int_{-\infty}^{+\infty} U_1(\xi, z) L(\xi) e^{i\frac{k}{2z}(x-\xi)^2} d\xi, \tag{4}$$

where ξ is the transversal coordinate at the lens principal plane, z is the distance from the front principal plane of the lens to the observation plane, and x is the transversal coordinate at the observation plane. This integral is easily solved considering $\int_{-\infty}^{+\infty} \exp\left[-\left(ax^2 + bx + c\right)\right] dx = \sqrt{\frac{\pi}{a}} \exp\left(b^2/4a - c\right)$, resulting in

$$U_2(x, z) = U_0 \sum_n a_n \sqrt{\frac{2\pi}{ik\left(1/f' - 1/z\right)}} e^{-i\frac{kx^2}{2\left(f'-z\right)}} e^{-i\frac{nqf'}{2k\left(f'-z\right)}\left(nqz - 2kx\right)}. \tag{5}$$

The intensity distribution is easily obtained as $I\left(x, z\right) = U_2(x, z) U_2^*(x, z)$, where * denotes complex conjugated,

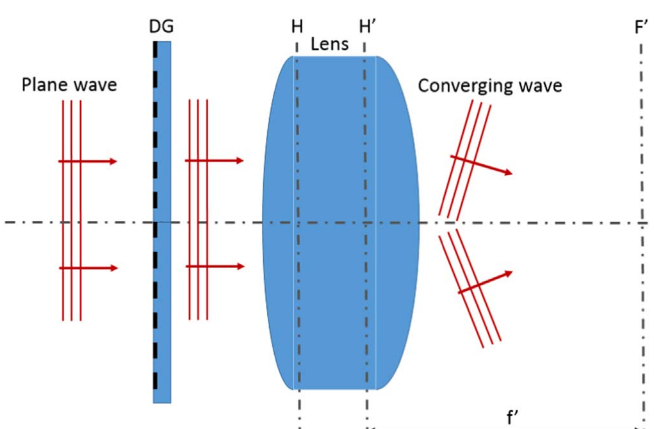

Fig. 1. Scheme of the proposed set-up for a convergent lens – similar for a divergent lens. DG is the diffraction grating, H, H' are the principal planes of the lens, F' is the front focal plane, and f' is the front focal length.

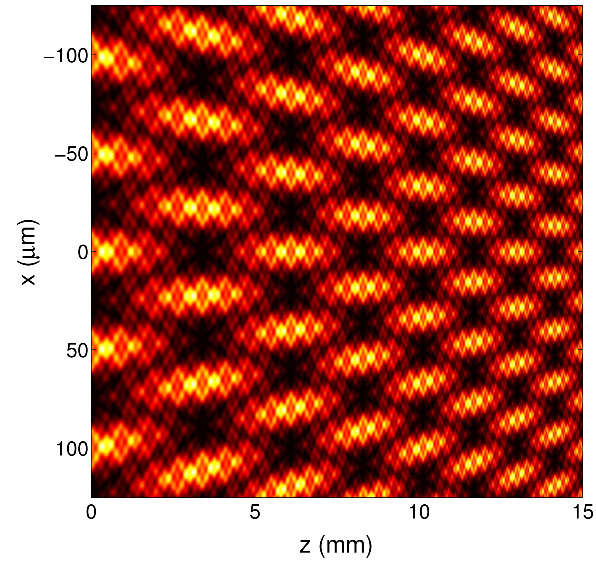

Fig. 2. Analytical near field intensity obtained with Eq. (6) by illuminating a Ronchi diffraction grating of period p=50 µm and a convergent lens of focal length $f' = 30$ mm with a collimated beam of wavelength $\lambda = 655$ nm.

$$I\left(x, z\right) = I_0 \sum_n \sum_{n'} a_n a_{n'}^* e^{i\frac{q\left(n-n'\right)x}{1-z/f'}} e^{-i\frac{q^2\left(n^2-n'^2\right)z}{2k\left(1-z/f'\right)}}, \tag{6}$$

where I_0 includes all constants that do not depend on the distance z. The first exponential factor is related to the period of the self-images, which depends on the distance, z, from the principal plane as

$$p' = p\left(1 - \frac{z}{f'}\right). \tag{7}$$

This dependence is linear and allows us to identify two points that correspond to the front focal plane, $p' = 0$, and the front principal plane of the lens, $p' = p$.

We show in Fig. 2 an example of intensity given by Eq. (6) with a collimated beam of wavelength $\lambda = 655$ mm, an amplitude diffraction Ronchi grating of period p=50 µm and a lens of focal length $f' = 30$ mm. As can be observed, Talbot self-images converge to the focal plane. It is important to notice that the period of the self-images must be calculated under paraxial approximation since the lens also curves the self-images far from the optical axis in a similar way to that is shown in [18,19]. Theoretically, the period has a linear dependence with z, $p' = az + b$. Then, after measuring the period of the fringes at each plane and calculating its linear dependence with z, the focal length, defined as the distance between the front principal plane and the focal plane, is obtained as

$$f' = -\frac{p}{a}. \tag{8}$$

The uncertainty in the focal length determination is given by $\Delta f'^2 = \left(1/a^2\right)\Delta p^2 + \left(p^2/a^4\right)\Delta a^2$. The period of commercial gratings is given with manufacture errors around 3 nm/m. Since we are measuring in a range of a few millimeters of the grating, we can consider the period of the grating well known and therefore Δp results negligible. Then, the uncertainty of the focal length is given by the uncertainty in the slope of the fitting of the measured periods [20]. At the end, the error in the focal length comes from errors committed in measuring the period of the fringes, p', and the corresponding distance z. The period of the fringes can be affected by misalignments of the optical system. Nevertheless, it can be calibrated previously since the period of the grating is well known. We can correct misalignments, simply measuring the period of the self-images without the lens. When the period of