Optik 156 (2018) 303–310

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Analytical model of the field of super-resolution focal spot based on sink-source model☆

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ARTICLE INFO

Article history:
Received 21 July 2017
Accepted 18 October 2017

Keywords:
Sink-source model
Point-spread function
Microscopy
Diffraction theory

ABSTRACT

The expressions of non-paraxial tightly focused vector beams propagating along a certain direction are given based on the sink-source model by using the spatiotemporal translation technology. The singularities on the beam waist plane are eliminated in the model. The field of a super-resolution focal spot with strong axial electric field component can be simulated by the model. The full width at half maximum (FWHM) of the intensity of focal spot where the field can be simulated by the model can be as small as 0.44 wavelength. The FWHM information of focal spot is preserved in the sink-source model. The refocusing phenomena of a focal laser spot by the 4f optical imaging system is also studied. The change of FWHM of an initial focal spot can be reflected from the refocused spot.

1. Introduction

The non-paraxial tightly focused vector expressions of laser beams have been extensively studied. The reason is that such laser beams are widely used in the fields of microscope technology and laser-material interaction. The complex point source model is often used to describe the tightly focused vector beams[1,2]. But in the beam model deduced from the complex point source model, there are singularities on the beam waist plane. Such a model isn't perfect. The expressions of non-paraxial tightly focused vector beams can be deduced from the sink-source model (SSM)[3–5]. The field model based on the SSM can simulate the field of a super-resolution focal spot with strong axial electric field component.

The point-spread function[PSF] is often applied to measure the resolving power of an optical system [6–8]. The standard derivation of the PSF is based on scalar theory and the paraxial approximation is insufficient for many high-resolution optical systems. The PSF where the source is an arbitrarily oriented electric dipole is often used as an evaluation method of the spatial resolution of an optical imaging system [9,10]. The smallest radiating electromagnetic unit is a dipole. Dipole waves are frequently used in physical optics to study focusing [11,12] or to describe image formation in optical microscopes [13,14] because of the scattered electric far field of small spherical objects. However, in the optical microscopes, especially in the laser scanning microscope, the laser beams propagate along a certain direction after passing through the focus area. Therefore, a fully vector model of the tightly focused light beam is needed to assess the spatial resolution of the laser scanning microscope.

☆ This work was supported by the Key Project for Innovation Research Groups of Guizhou Provincial Department of Education, China (Grant No. KY[2016]030, No. KY[2016]028), the Science and Technology Foundation of Guizhou Province, China (Grant No. (2014)2090).

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https://doi.org/10.1016/j.ijleo.2017.10.077
0030-4026/

A fully vector model of PSF can be established by using the electric dipole method [14,15]. However, there is a singularity for the field at the location of the dipole, a small positive number is intentionally added to the radius vector of the field spot to remove this singularity. Thus the field of an infinitely small focal spot is simulated by the PSF based on the dipole model. The FWHM information of the intensity of focal spot is lost. A vector model based on SSM can provide the PSF of a focus spot with strong axial field component. The FWHM information of spot is preserved in the SSM and can be reflected in the refocusing process. Another problem is how to simulate a fully vector field propagating in a certain direction. The problem can be solved by using the spatiotemporal translation technology [16].

In this paper, the analytical model of field based on SSM is studied. The model of tightly focused vector beam [17] is used to simulate the light beam in the laser microscopy system. The refocusing of the tightly focused vector beam under the non-paraxial approximation in a 4f optical imaging system is studied.

2. Theory

2.1. The sink-source model and the dipole model

The sink-source model can be found in papers [4,5,17]. The expressions of sub-cycle pulsed focused vector beams are presented in papers [5,17]. Two opposite charges oscillating against each other are considered. They can be described as an oscillating dipole located at the origin of the coordinate system

$$P(r,t) = p(t)\delta(r)\overrightarrow{e_z}, \tag{1}$$

where $p(t) = p_0 e^{i\omega t}$.

In order to simulate the axial electric field polarization of a tightly focused vector beam, the polarization of the dipole is in the z direction. p_0 is the dipole moment. The oscillating dipole emits a spherically outgoing electromagnetic wave. A focused beam propagating in a certain direction, for which we take the z direction, can be obtained by the sink-source method. The source dipole is moved from the origin to a complex position along the z axis in the sink-source method. It is a very useful mathematical technique to obtain focused propagating solutions of Maxwell's equations. The spatiotemporal translation is introduced as

$$\mathrm{z} \to \mathrm{z}' = \mathrm{z} + i\mathrm{z}_0\,,\ \mathrm{t} \to \mathrm{t}' = t - t_0 + i\mathrm{z}_0/c, \tag{2}$$

where $z_0 = kw_0^2/2$, and w_0 is the beam waist parameter. The spatial coordinate z and the time ct are shifted by the same imaginary amount. We introduce the complex distance and the complex retarded time

$$R' = \sqrt{x^2 + y^2 + (z + iz_0)^2},\ \tau' = t' - \mathrm{R}'/\mathrm{c}\,. \tag{3}$$

In this way, on axis (for $x = y = 0$) the shift cancels from the retarded time, and we will be able to retrieve the standard radially polarized beam in the paraxial approximation.

In the sink-source method, the source is always accompanied with a sink. The sink can absorb the fields coming in from the outer boundary. Such a method can be viewed as another expression of Huygens-Fresnel Principle. In order to get the expressions of the light field, a δ function is given as

$$D\left(R', t'\right) = \frac{c^2\mu_0}{4\pi}\frac{\delta\left(t' - R'/c\right) - \delta\left(t' + R'/c\right)}{R'}, \tag{4}$$

where μ_0 is the magnetic permeability of vacuum. The expression (4) shows a δ spherical pulse wave. The pulse converges toward the coordinate origin at the time $t' < 0$, then diverges from the coordinate origin at the time $t' > 0$.

By folding the source with the expression (4), the expression of the dipole moment in SSM can be written as

$$\overrightarrow{P} = (p_1 - p_2)\overrightarrow{e_z} = (p_0 e^{i\omega(t + i\frac{z_0}{c} - \frac{R'}{c})} - p_0 e^{i\omega(t + i\frac{z_0}{c} + \frac{R'}{c})})\overrightarrow{e_z},$$

where $p_1 = p_0 e^{i\omega(t + i\frac{z_0}{c} - \frac{R'}{c})}$ and $p_2 = p_0 e^{i\omega(t + i\frac{z_0}{c} + \frac{R'}{c})}$.

Now the electric Hertz vector in SSM can be written as

$$\overrightarrow{\prod}_e = \frac{c^2\mu_0}{4\pi R'}(p_1 - p_2)\overrightarrow{e_z} = \frac{c^2\mu_0}{4\pi R'}(p_0 e^{i\omega(t + i\frac{z_0}{c} - \frac{R'}{c})} - p_0 e^{i\omega(t + i\frac{z_0}{c} + \frac{R'}{c})})\overrightarrow{e_z}, \tag{5}$$

The magnetic Hertz vector $\overrightarrow{\prod}_m$ is zero. By solving the Maxwell's equations, the expressions of light field can be obtained as [18]

$$\vec{E} = \nabla \times \left(-\frac{1}{\mathrm{c}}\dot{\overrightarrow{\Pi}}_m + \nabla \times \overrightarrow{\Pi}_e\right) + \left(\nabla^2 \overrightarrow{\Pi}_e - \frac{1}{c^2}\ddot{\overrightarrow{\Pi}}_e\right), \tag{6}$$