# An efficient hysteresis modeling methodology and its implementation in field computation applications

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ABSTRACT

Field computation in media exhibiting hysteresis is crucial to a variety of applications such as magnetic recording processes and accurate determination of core losses in power devices. Recently, Hopfield neural networks (HNN) have been successfully configured to construct scalar and vector hysteresis models. This paper presents an efficient hysteresis modeling methodology and its implementation in field computation applications. The methodology is based on the application of the integral equation approach on discretized triangular magnetic sub-regions. Within every triangular sub-region, hysteresis properties are realized using a 3-node HNN. Details of the approach and sample computation results are given in the paper.

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## 1. Introduction

Electromagnetic field computation in nonlinear media and media exhibiting hysteresis is, in general, a crucial activity to a wide variety of applications such as those involving analysis of magnetic recording processes and estimation of core losses in power devices (see, for instance, to [1–8]). In the past, the integral equations (IE) approach has emerged as one of the most efficient methodologies to carry out computations in media exhibiting hysteresis [9,10]. Recently, Hopfield neural networks (HNN) have been successfully configured to construct scalar and vector hysteresis models [11–13]. This paper presents an efficient hysteresis modeling methodology and its implementation in field computation applications. In specific, the paper extends previous modeling approaches to the triangular sub-region case and presents an IE implementation scheme for hysteretic media. Details of the approach as well as some simulation results are given in the following sections.

## 2. The efficient triangular hysteresis modeling approach and field computation methodology

In general, Hopfield neural networks are single-layer feedback networks which are fully connected. Each node is connected to other nodes through a connection weight. Node outputs evolve with time so that the network converges towards the minimum of the network's formulated quadratic energy function [14]. Given the success in constructing elementary hysteresis operators using HNNs, it can be demonstrated that a triangular sub-region exhibiting hysteresis may be modeled by a 3-node HNN with positive feedbacks, as shown in Fig. 1(a) [11–13]. In this configuration, the activation function suggested in [13] is adopted and the direction of every node is identified by the line joining the center point to its corresponding vertex. The positive feedbacks $k_{i,j}$ between different triangle nodes $i$ and $j$ may be given by the expression:

$$k_{i,j} = k_{copl}|\cos(\phi_{i,j})|, k_{copl} \leqslant 1, \tag{1}$$

where, $\varphi_{ij}$ is the angle subtended between nodes $i$ and $j$ orientations as shown in Fig. 1(b).

Using the Stoner-Wohlfarth-like activation proposed in [13], the HNN activation function $f(x)$ may be expressed by:

$$f(x) = cf_c(x) + (1 - c)f_d(x), c \leqslant 1, \tag{2}$$

where, $f_c(x)$ and $f_d(x)$ are the sigmoid continuous and signum discrete activation functions, respectively, given by:

$$f_c(x) = \tanh(ax), a > 0, \tag{3}$$

$$f_d(x) = \{+1 \text{ if } x > 0; -1 \text{ if } x < 0; \text{unchanged if } x = 0\} \tag{4}$$

It should be pointed out that the width of a hysteresis loop expressed by (2) would, generally, be equivalent to $2(1 - c)k_{copl}$ while its squareness may be controlled by the values of $a$ and $c$ (please refer to [13]).

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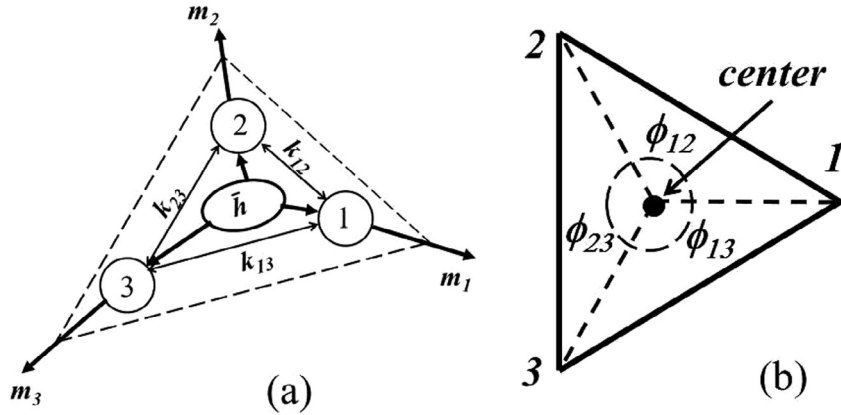  E-mail address: adlyamr@gmail.com (A.A. Adly).

**Fig. 1.** (a) Realization of a vector hysteresis operator representing a triangular sub-region using a 3-node HNN, and (b) example of a triangular sub-region showing angles subtended between different nodes.

Assuming an external input $\bar{h}$, the activation rule for any arbitrary node $i$ and the node output $m_i$ may be deduced from:

$$m_i(t+1) = cf_c(net_i(t)) + (1-c)f_d(net_i(t)), \text{ where } net_i(t)$$
$$= \bar{h}(t) \cdot \left(\frac{\bar{m}_i}{m_i}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{3} k_{i,j} m_j(t), \tag{5}$$

where the output $m_i$ orientation of any arbitrary node $i$ is as shown in Fig. 1.

As per HNN internal mechanism [14], the network converges to the minimum state for the energy given by:

$$E = -\sum_{i=1}^{3} \left\{ \bar{h} \cdot \bar{m}_i + \frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^{3} k_{i,j} m_i m_j \right\}, \quad \bar{m} = \sum_{i=1}^{3} \bar{m}_i \tag{6}$$

Fig. 2 demonstrates sample scalar and rotational simulations corresponding to different triangle configurations.

Assuming constant magnetization within every triangular sub-division and assuming that within each triangle the applied field is equivalent to its value at the triangular sub-division center coordinates, the total field $H_u$ acting on the $u^{th}$ triangle may be given by:

$$\bar{H}_u = \bar{H}app_u + \sum_{v=1, v \neq u}^{N} -\nabla \int_{-\infty}^{+\infty} \left(\frac{\bar{m}_v \cdot \bar{R}_{vu}}{4\pi |\bar{R}_{vu}|^3}\right) S_v dz, \tag{7}$$

where, $N$ is the total number of triangular sub-divisions, $H_{app}$ is the externally applied field, $S_v$ is the $v^{th}$ triangle area, while $\bar{R}_{vu}$ is the distance vector extending between center points of triangles $u$ and $v$.
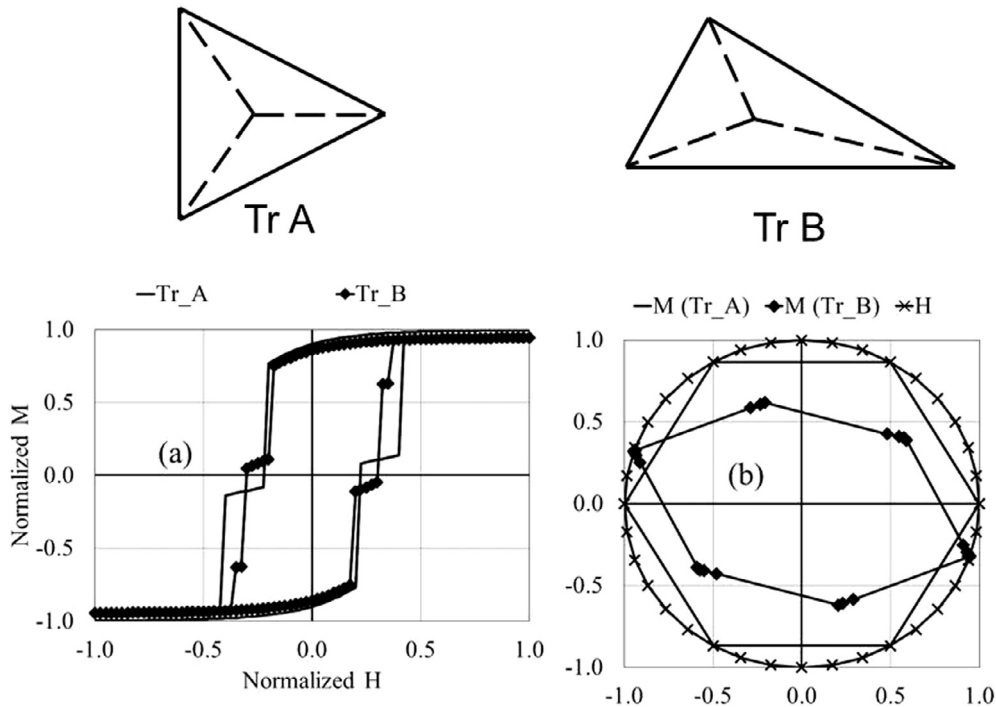


**Fig. 2.** (a) Normalized scalar $M$-$H$ curves, and (b) Normalized rotational $M$-$H$ curves for $k_{copl} = 0.3$, $a = 3$, $c = 0.7$ and corresponding to two triangles (Tr_A and Tr_B) having angles subtended between centers and vertices 0°, +120°, −120° and −15.94°, +116.56°, −158.2°, respectively.