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Reduced order modeling strategies for computational multiscale fracture

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Highlights

- The RVE homogenization is stated in terms of a saddle-point problem.
- RVE unknowns are unconventionally rephrased in terms of the fluctuation strains.
- Separate Reduced Order Models are used in the RVE elastic and inelastic domains.
- HPROM combines Reduced Order Modeling and Reduced Optimal Quadrature techniques.
- Results exhibit outstanding speedups for multi-scale fracture problems.

Abstract

The paper proposes some new computational strategies for affordably solving multiscale fracture problems through a FE^2 approach. To take into account the mechanical effects induced by fracture at the microstructure level the Representative Volume Element (RVE), assumed constituted by an elastic matrix and inclusions, is endowed with a large set of cohesive softening bands providing a good representation of the possible microstructure crack paths. The RVE response is then homogenized in accordance with a model previously developed by the authors and upscaled to the macro-scale level as a continuum stress–strain constitutive equation, which is then used in a conventional framework of a finite element modeling of propagating fracture.

For reduced order modeling (ROM) purposes, the RVE boundary value problem is first formulated in displacement fluctuations and used, via the Proper Orthogonal Decomposition (POD), to find a low-dimension space for solving the reduced problem. A *domain separation strategy* is proposed as a first technique for model order reduction: unconventionally, the low-dimension space is spanned by a basis in terms of fluctuating strains, as primitive kinematic variables, instead of the conventional formulation in terms of displacement fluctuations. The RVE spatial domain is then decomposed into a *regular domain* (made of the matrix and the inclusions) and *a singular domain* (constituted by cohesive bands), the required RVE boundary conditions are rephrased in terms

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of strains and imposed via Lagrange multipliers in the corresponding variational problem. Specific low-dimensional strain basis is then derived, independently for each domain, via the POD of the corresponding strain snapshots.

Next step consists of developing a hyper-reduced model (HPROM). It is based on a second proposed technique, the *Reduced Optimal Quadrature* (ROQ) which, again unconventionally, is determined through optimization of the numerical integration of the primitive saddle-point problem arising from the RVE problem, rather than its derived variational equations, and substitutes the conventional Gauss quadrature. The ROQ utilizes a very reduced number of, optimally placed, sampling points, the corresponding weights and placements being evaluated through a greedy algorithm. The resulting low-dimensional and reduced-quadrature variational problem translates into very relevant savings on the computational cost and high computational speed-ups.

Particular attention is additionally given to numerical tests and performance evaluations of the new hyper-reduced methodology, by “a-priori” and “a-posteriori” error assessments. Moreover, for the purposes of validation of the present techniques, a real structural problem exhibiting propagating fracture at two-scales is modeled on the basis of the strain injection-based multiscale approach previously developed by the authors. The performance of the proposed strategy, in terms of speed-up vs. error, is deeply analyzed and reported.

Keywords: Reduced Order Modeling (ROM); Hyper-Reduced Order Modeling (HPROM); Multiscale fracture models; Computational homogenization; Reduced Optimal Quadrature (ROQ)

Acronyms

BVP: Boundary Value Problem
CSDA: Continuum-Strong Discontinuity Approach
EFEM: Embedded Finite Element Methodology
FE^2: Two-scale (macro and micro or meso) model, where both scales of analysis are represented by finite element approaches
HF: High Fidelity model
HPROM: HyPer-Reduced Order Modeling
OQN: Optimal Quadrature Number
POD: Proper Orthogonal Decomposition
ROM: Reduced Order Modeling
ROQ: Reduced Optimal Quadrature
RVE: Representative Volume Element
SVD: Singular Value Decomposition
VBVP: Variational Boundary Value Problem

Symbols

Number of dimensions of vectors or vectorial spaces

n_σ: Number of components of the stress and strain (or micro-stress and micro-strain) vectors described with Voigt's notation, typically: for strains and stresses in plane states: $n_\sigma = 4$, and $\boldsymbol{\varepsilon} = [\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}]^T$ or $\boldsymbol{\sigma} = [\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}]^T$.

n_ε: Dimension of the reduced micro-strain fluctuation space (number of basis of $\boldsymbol{\Psi}$ spanning the full space of reduced micro-strains).

$n^I_{\varepsilon,reg}$: Dimension of the reduced micro-strain fluctuation space $\boldsymbol{\Psi}^I_{reg}$ associated with the inelastic snapshots and Gauss points in the regular domain ($\mathcal{B}_{\mu,\mathrm{reg}}$).

$n^I_{\varepsilon,coh}$: Dimension of the reduced micro-strain fluctuation space $\boldsymbol{\Psi}^I_{coh}$ associated with the inelastic snapshots and Gauss points in the domain of cohesive bands ($\mathcal{B}_{\mu,\mathrm{coh}}$).

n_φ: Dimension of the reduced micro internal energy space (number of basis of $\boldsymbol{\Phi}$ spanning the full space of reduced micro-internal energy φ_μ).