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Model uncertainties in computational viscoelastic linear structural dynamics

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Abstract

This paper deals with the analysis of the propagation of uncertainties in computational linear dynamics for linear viscoelastic composite structures in the presence of uncertainties. In the frequency domain, the generalised damping matrix and the generalised stiffness matrix of the stochastic computational reduced-order model are random frequency-dependent matrices. Due to the causality of the dynamical system, these two frequency-dependent random matrices are statistically dependent and their probabilistic model involves a Hilbert transform. In this paper, a computational analysis of the propagation of uncertainties is presented for a composite viscoelastic structure in the frequency range.

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Keywords: Uncertainty quantification, Viscoelastic, Nonparametric probabilistic approach, Structural dynamics, Hilbert transform, Kramers-Kronig relations, Reduced-order model

1. Introduction

In structural engineering, uncertainties have to be accounted for the design and the analysis of a structure using computational models. In the computational models, the sources of uncertainties are due to the model-parameters uncertainties, as well as the model uncertainties induced by modelling errors. In the probabilistic framework, uncertainty quantification has extensively be developed in the last two decades (see for instance [1–3]).

The objective of this paper is to present the numerical analysis of an extension (recently proposed in [4–6]) of the nonparametric probabilistic approach of uncertainties [7] in computational linear structural dynamics for viscoelastic composite structures in the frequency-domain. In the framework of linear viscoelasticity (see for instance [8,9]) and in the frequency domain, the generalised damping matrix $[\mathcal{D}(\omega)]$ and the generalised stiffness matrix $[\mathcal{K}(\omega)]$ of the reduced-order computational model depend on frequency ω. The nonparametric probabilistic approach of uncertainties consists in modelling this two frequency-dependent generalised matrices by frequency-dependent random

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matrices $[\mathbf{D}(\omega)]$ and $[\mathbf{K}(\omega)]$ respectively. However, as these two matrices come from a causal dynamical system, the causality implies two compatibility equations, also known as the Kramers-Kronig relations [10,11], involving the Hilbert transform [12]. A summary of the construction of the deterministic reduced-order computational model is presented in Section 2. Section 3 deals with the construction of the nonparametric probabilistic model using the Hilbert transform. In Section 4 a numerical example is presented.

2. Computational model in linear viscoelasticity

2.1. Linear viscoelastic constitutive equation

Let $\Omega = \Omega_e \cup \Omega_{ve}$ be an open, connected, and bounded domain of $\mathbb{R}^3$, constituted of two parts Ω_e and Ω_{ve}. The first part Ω_e is occupied by a purely elastic medium while the second part Ω_{ve} is occupied by a linear viscoelastic medium. In a cartesian frame $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, let $\mathbf{x} = (x_1, x_2, x_3)$ be the position vector of any point in Ω. Let $\mathbf{u}(\mathbf{x}, t)$ be the displacement field defined on Ω. The linearised strain tensor is denoted by $\{\varepsilon_{kh}\}_{hk}$ and the Cauchy stress tensor by $\{\sigma_{ij}\}_{ij}$, with i, j, k, and h in $\{1, 2, 3\}$. The theory of linear viscoelasticity is used in order to obtain the constitutive equation of the viscoelastic medium occupied by domain Ω_{ve}. For $t \leq 0$, the system is assumed to be at rest. In the time domain, the constitutive equation is then written as

$$\sigma(\mathbf{u}(\mathbf{x}, t)) = \int_0^t \mathcal{G}(\mathbf{x}, \tau) : \varepsilon(\dot{\mathbf{u}}(\mathbf{x}, t - \tau)) d\tau \,, \tag{1}$$

in which $\dot{\mathbf{u}}$ is the partial derivative of $\mathbf{u}$ with respect to t, where $t \mapsto \mathcal{G}(\mathbf{x}, t)$ is the relaxation function defined on $[0 + \infty[$ with values in the fourth-order tensor that satisfies the usual symmetry properties. Function $t \mapsto \mathcal{G}(\mathbf{x}, t)$ is differentiable with respect to t on $]0, +\infty[$ and its partial time derivative $t \mapsto \{\dot{\mathcal{G}}_{ijkh}(\mathbf{x}, t)\}_{ijkh}$ is assumed to be integrable on $[0\,, +\infty[$. At time $t = 0$, the initial elasticity tensor $\mathcal{G}(\mathbf{x}, 0)$ is positive definite. Consequently, Eq. (1) can be rewritten as

$$\sigma(\mathbf{u}(\mathbf{x}, \mathbf{t})) = \mathcal{G}(\mathbf{x}, 0) : \varepsilon(\mathbf{u}(\mathbf{x}, t)) \; + \int_{-\infty}^{+\infty} g(\mathbf{x}, \tau) : \varepsilon(\mathbf{u}(\mathbf{x}, t - \tau)) dt \,, \tag{2}$$

where fourth-order tensor $g(\mathbf{x}, t)$ is defined by $g(\mathbf{x}, t) = 0$, if $t < 0$ and $g(\mathbf{x}, t) = \dot{\mathcal{G}}(\mathbf{x}, t)$ if $t \geq 0$. Taking the Fourier transform with respect to t of both sides of Eq. (2), and introducing the real part $\widehat{g}^R(\mathbf{x}, \omega) = \Re e\{\widehat{g}(\mathbf{x}, \omega)\}$ and the imaginary part $\widehat{g}^I(\mathbf{x}, \omega) = \Im m\{\widehat{g}(\mathbf{x}, \omega)\}$, the constitutive equation in the frequency domain can be written as

$$\sigma(\widehat{\mathbf{u}}(\mathbf{x}, \omega)) = (a_0(\mathbf{x}) + a(\mathbf{x}, \omega) + \mathrm{i}\, \omega\, b(\mathbf{x}, \omega)) : \varepsilon(\widehat{\mathbf{u}}(\mathbf{x}, \omega)) \,, \tag{3}$$

where $a_0(\mathbf{x}) = \mathcal{G}(\mathbf{x}, 0)$ and where the components $a_{ijkh}(\mathbf{x}, \omega)$ and $b_{ijkh}(\mathbf{x}, \omega)$ of the fourth-order real tensors $a(\mathbf{x}, \omega)$ and $b(\mathbf{x}, \omega)$ are the viscoelastic coefficients that are such that

$$a(\mathbf{x}, \omega) = \widehat{g}^R(\mathbf{x}, \omega) \quad , \quad \omega\, b(\mathbf{x}, \omega) = \widehat{g}^I(\mathbf{x}, \omega) \,. \tag{4}$$

Since g is a causal function of time, the real part $\widehat{g}^R$ and imaginary part $\widehat{g}^I$ of its Fourier transform $\widehat{g}$ are related through a set of compatibility equations also known as the Kramers-Kronig relations [10,11]. These relations involve the Hilbert transform [12] and are written as

$$\widehat{g}^R(\mathbf{x}, \omega) = \frac{1}{\pi}\, p.v \int_{-\infty}^{+\infty} \frac{\widehat{g}^I(\mathbf{x}, \omega')}{\omega - \omega'}\, d\omega', \quad \widehat{g}^I(\mathbf{x}, \omega) = -\frac{1}{\pi}\, p.v \int_{-\infty}^{+\infty} \frac{\widehat{g}^R(\mathbf{x}, \omega')}{\omega - \omega'} d\omega', \tag{5}$$

in which $p.v$ denotes the Cauchy principal value. From Eqs. (4) and (5), the following relation between the viscoelastic tensors $a(\mathbf{x}, \omega)$ and $b(\mathbf{x}, \omega)$ can then be deduced, for all $\omega > 0$,

$$a(\mathbf{x}, \omega) = \frac{\omega}{\pi}\; p.v \int_{-\infty}^{+\infty} \frac{b(\mathbf{x}, \omega')}{\omega - \omega'}\, d\omega' \,. \tag{6}$$