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The computational modeling of problems on domains with small holes

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Abstract

The modeling challenges arising when the problem domain has small supported holes in it are considered through a representative membrane problem. Such problems are sometimes modeled intuitively in engineering practice by taking the limiting case of holes with zero radius. This intuitive model is incorrect, since it has no mathematical solution. It is demonstrated, however, that finite element approximations based on it can still satisfy verification tests and appear to converge, leading to erroneous recovery of quantities of interest. This points to the need for an alternate approach where the holes of finite radius are properly incorporated in the modeling, and robustness with respect to the radius is maintained. To this end, a computational method is presented which combines analytic knowledge of the solution singularities with finite element approximation of its smooth components. Theoretical and numerical results are provided, establishing the efficacy and robustness of the method in extracting quantities of interest. The method converges both with respect to the size of the holes and the mesh discretization parameter, and provides a more accurate alternative to using the asymptotic limit.

Keywords: Finite elements, logarithmic singularities, prestressed membrane, electrostatics, perforated domain, verification, validation

1. Introduction

We consider the flat prestressed membrane Ω shown in Figure 1, attached to supporting discs ω_i, $i = 1, 2, \ldots n$, with distributed normal stress g acting on its boundary $\partial\Omega$ (i.e. the membrane has Dirichlet conditions on $\partial\omega_i$ and Neumann conditions on $\partial\Omega$). Our goal in this paper is to analyze two different approaches to modeling this problem in the case that the radii σ_i of the discs ω_i are *small*. We address both the choice of underlying mathematical model and of the method of discretization. Our results apply to several mathematically equivalent problems such as heat transfer in the presence of

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