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Gain and loss of esteem, direct reciprocity and Heider balance

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HIGHLIGHTS

- We prove that for asymmetric relations, jammed states are generic on the contrary to the symmetric version.
- As a consequence, the balance is not attained in most cases.
- The mechanism of gain and loss of esteem, as described by Aronson, is built into model equations for the first time.
- A new phase diagram is worked out, with three phases: jammed, HB and paradise.

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ABSTRACT

The effect of gain and loss of esteem is introduced into the equations of time evolution of social relations, hostile or friendly, in a group of actors. The equations allow for asymmetric relations. We prove that in the presence of this asymmetry, the majority of stable solutions are jammed states, i.e. the Heider balance is not attained there. A phase diagram is constructed with three phases: the jammed phase, the balanced phase with two mutually hostile groups, and the phase of so-called paradise, where all relations are friendly.

1. Introduction

Despite the eternal doubts whether social phenomena can be described quantitatively [1–3], mathematical modeling of interpersonal relations has long tradition [4–10]. The idea is tempting: a bottom-up path from understanding to control and predict our own behavior seems to promise a higher level of human existence. On the other hand, any progress on this path is absorbed by the society in a kind of self-transformation, what makes the object of research even more complex. As scholars belong to the society, an observer cannot be separated from what is observed; this precludes the idea of an objective observation. Yet, for scientists, the latter idea is paradigmatic; their strategy is to conduct research as usual. As a consequence, the hermeneutically oriented multi-branched mainstream is accompanied by a number of works based on agent-based simulations, statistical physics and traditional, positivist sociology.

Theory of the Heider balance [5] is one of such mathematical strongholds in the body of social science. Based on the concept of the removal of cognitive dissonance (RCD) [4], it has got a combinatorial formulation in terms of graph theory [11].

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In a nutshell, the concept is as follows: interpersonal relations in a social network are either friendly or hostile. The relations evolve over time as to implement four rules: friend of my friend is my friend, friend of my enemy is my enemy, enemy of my friend is my enemy, enemy of my enemy is my friend. In a final 'balanced' state, the cognitive dissonance is absent [11,12]. There, the network is divided into two parts, both internally friendly and mutually hostile. A special case when all relations are friendly (so-called paradise) is also allowed. More recently, Monte-Carlo based discrete algorithms have been worked out to simulate the dynamics of the process of RCD on a social network [13]. In parallel, a set of deterministic differential equations has been proposed as a model of RCD [14–16]. This approach has been generalized to include asymmetric relations [17] as well as the mechanism of direct reciprocity [18], which was supposed to remove the asymmetry.

Our aim here is to add yet another mechanism, i.e. an influence of the rate of the change of relations to the relations themselves. This mechanism has been described years ago by Elliot Aronson and termed as the gain and loss of esteem [19]; see also the description and literature in [20]. Briefly, an increase of sympathy x_{ij} of an actor i about another actor j appears to be an independent cause of sympathy x_{ji} of the actor j about the actor i. By 'independent' we mean: not coming from x_{ij}, but from the time derivative $\mathrm{d}x_{ij}/\mathrm{d}t$. In summary, both the relation x_{ij} itself and its time derivative influence the relation x_{ji}. The efficiencies of these impacts and the rate of RCD play the roles of parameters in our model. We note that the concept of gain and loss of esteem has triggered a scientific discussion which is not finished until now [21–25]. Among implications, let us mention two: for man–machine cooperation [26] and for evaluations of leaders as dependent on the time evolution of their behavior (the so-called St. Augustine effect) [27]. In our opinion, it is worthwhile to try to include the effect in the existing theory of RCD.

Here we are interested in three phases of the system of interpersonal relations: the jammed phase, the balanced phase with two mutually hostile groups, and the phase of so-called paradise, where all relations are friendly. The two latter phases are known from early considerations of the Heider balance in a social network [11]. The jammed phase is the stationary state of relations, where the Heider balance is not attained. Jammed states have been discussed for the first time for the case of symmetric relations (i.e. $x_{ji} = x_{ij}$) by Tibor Antal and coworkers [13]. The authors have shown, that this kind of states appear rather rarely, and the probability of reaching such a state decreases with the system size. A similar conclusion has been drawn also for the evolution ruled by the differential equations [28]. Both these proofs rely on the assumption that the matrix of relations is symmetric.

Our goal here is twofold. First, in Section 2 (Section 2.3) we provide a proof that with asymmetric relations driven by differential equations, the number of jammed states produced by RCD is at least N times larger than the number of balanced states, where N is the number of nodes. Second, in Section 3 we introduce the mechanism of the gain and loss of esteem [19], unifying the model approach with previous work [18]. The new effect is discussed within the frames of differential equations for asymmetric relations, together with RCD and the direct reciprocity. In Section 4 we construct a phase diagram, with the model parameters as coordinates, and the ranges of parameters are identified where the three above states appear. Final remarks close the text.

2. The jammed states

2.1. Discrete algorithm for symmetric relations: $N = 9$

In [13], a discrete algorithm (the so-called Constrained Triad Dynamics) has been proposed to model RCD. For each pair of nodes (ij) of a fully connected graph, an initial sign $x_{ij} = \pm 1$ (friendly or hostile) is assigned to the link between nodes i and j. For this initial configuration, the number of imbalanced triads (ijk) (such that $x_{ij}x_{jk}x_{ki} < 0$) is calculated. This number can be seen as an analogue to energy; let us denote it as U. The evolution goes as follows. A link is selected randomly. If the change of its sign lowers U, the sign is changed; if U increases, the change is withdrawn; if U remains constant, the sign is changed with probability 1/2. Next, another link is selected, and so on. As a consequence, in a local minimum of U the system cannot evolve; the state is either balanced or jammed.

According to [13], the minimal network size where jammed states are possible is 9 nodes. An example of jammed states for $N = 9$ is the case of three separate triads, where each link within each triad is positive (friendly) and each link between nodes of different triad is negative (hostile). As shown in [13], each modification of a link leads to an increase of the number of unbalanced triads above the minimal value $U = 27$. This can be seen easily when we notice that for each unbalanced triangle, each its vertex is in different triad; hence the number of these triangles is $3 \times 3 \times 3 = 27$. A simple inspection tells us that a change of any friendly link to hostile state enhances energy to $U = 34$, while any opposite change gives $U = 28$; hence the configuration is stable.

2.2. Continuous algorithm for symmetric relations: $N = 9$

The same jammed state appears for RCD described by the differential equations [14]

$$\frac{\mathrm{d}x_{ij}}{\mathrm{d}t} = H(1 - |x_{ij}|) \sum_{k}^{N-2} x_{ik}x_{kj} \tag{1}$$