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Determining the utility functions of criteria used in the evaluation of real estate

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ABSTRACT

This article presents an application of the UTA method and its variant UTA-CR to determining utility functions for the multicriteria evaluation of residential real estate. Data for the city of Volta Redonda, Brazil, were used in this study. Unlike UTA, UTA-CR makes use of the decision agents' preferences in relation to a set of criteria to determine a ranking of the alternatives. It was concluded that UTA-CR manages to obtain utility functions closer to the preferences of the decision agents as compared to these that result from the use of UTA. This demonstrates an important advantage of UTA-CR over UTA.

1. Introduction

This article deals with obtaining utility functions for criteria used in the evaluation of residential real estate. The UTA method (Jacquet-Lagrèze and Siskos, 1982; Siskos et al., 2004) and its variant UTA-CR (Rangel, 2002) were used in the study related here with the aim of evaluating a selected subset of residential properties which are available for rent in the Municipality of Volta Redonda in the south of the state of Rio de Janeiro, Brazil.

In order to be implemented, the UTA method needs an a priori piece of information: the ranking of the alternatives or a subset of these alternatives present in the process. The ranking supplied by the decision agents is used as a restriction of a linear programming problem (LPP), which has its own objective function, the minimizing of the sum of the errors associated with the ranking of each alternative, that is, the value of the global utility of each alternative.

The variant of the UTA method called UTA-CR is used in the same way as the original UTA method. This variant seeks to obtain utility functions as close as possible to the decision agents' preferences. In order to obtain the new utility functions of the criteria it asks the decision agents to express their preferences in relation to the set of criteria, and not in relation to the alternatives as occurs when the original UTA method is used. A multicriteria decision support method of ranking is, thus, used and the ranking of the alternatives which will be used in the implementation of the UTA-CR method is determined, in the same way as when the original UTA method is used. In order to obtain utility functions as close as possible to the decision agents' preferences a new mathematical model was developed, presenting a new objective function and new restrictions.

In the study presented in this article, instead of asking a specialist for a subjective ranking of the properties present in the process to serve as input data in the implementation of the UTA method and its UTA-CR variant, a ranking of the properties obtained by using the TODIM method (Gomes and Lima, 1992; Gomes and Rangel, 2007) was employed.

The implementation of the two methods was carried out with the aim of checking the differences between the utility functions obtained by the original UTA method and by its variant UTA-CR. The results of the two implementations are presented and discussed later in this article.

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2. Presentation of the UTA and UTA-CR methods

2.1. The UTA method

The UTA method (Jacquet-Lagrèze and Siskos, 1982) has the same axiomatic bases underlying the multiattribute utility theory (MAUT) (Fishburn, 1970; Keeney and Raiffa, 1993) and is used to determine the utility functions of the criteria (Belton and Stewart, 2002; Roy and Bouyssou, 1993).

This method uses linear programming to determine the values of the variables and, thus, obtain utility functions. The objective function of the model then seeks to minimize the sum of the errors associated with the global value of the alternatives so as to respect the a priori preferences proposed. In this way, the ordinal regression problem that is dealt with by the UTA method is the following: "Having a preference structure of weak order $R(\succ,\sim)$, with "$\succ$" signifying strict preference and "$\sim$" indifference in a set of alternatives or actions, the adjustment of the additive utility function based on multiple criteria is obtained in such a way that the resulting preference structure is as consistent as possible with the initial structure" Jacquet-Lagrèze and Siskos (1982) proposed a priori by the decision agents.

Consider the set of alternatives A, which is evaluated by a set of criteria $g=(g_1,g_2,g_3,\ldots,g_n)$, with n being the number of criteria and g_i the performance in criterion i. The aggregation of all the criteria into a single criterion is performed through the building of a multiattribute utility function which is represented as $U(g)=U(g_1,g_2,g_3,\ldots,g_n)$. The relationship of strict preference between two alternatives is called P and the indifference relationship I (Roy and Bouyssou, 1993; Vincke, 1989), respectively.

In a multicriteria problem, the viable alternatives are evaluated by a set of criteria defined by the evaluators, with the aim of analyzing the characteristics of the alternatives. In the additive model, when only using one criterion, the preference between the alternatives, among the set of alternatives A, can be expressed as follows:

$$a\succ b \Leftrightarrow g_i(a)>g_i(b), \tag{1}$$

$$a\sim b \Leftrightarrow g_i(a)=g_i(b), \tag{2}$$

which means that this criterion defines in set A a relationship of the ranking $(\succ,\sim)$ of the alternatives.

If $g(a)=[g_1(a),g_2(a),g_3(a),\ldots,g_n(a)]$ is the multicriteria evaluation of an alternative, then the following properties of the multiattribute utility function U, in relation to the set of alternatives A is observed:

$$U[g(a)]>U[g(b)]\Leftrightarrow aPb, \tag{3}$$

$$U[g(a)]=U[g(b)]\Leftrightarrow aIb, \tag{4}$$

and the relationship $R=P\cup I$ defines a weak order of the alternatives.

The utility function is additive when it has the form below

$$U[g]=\sum_{i=1}^{n}u_i(g_i), \tag{5}$$

where each $u_i(g_i)$ is the marginal utility of the performance g_i in the criterion i. One fundamental hypothesis that needs to be respected when applying an additive utility function is the condition of mutual independence of the criteria in function of the preferences (Keeney and Raiffa, 1993; Zopounidis and Dimitras, 1998).

Consider $g_i{}^*$ and g_{i^*} as the upper and lower limits of each criterion i present in a decision making problem. Assuming a non-decrease of the preferences in each criterion, then the marginal utilities, u_i, are increasing or decreasing monotonous functions. Therefore, the utility functions can be normalized inside the interval [0,1], obtaining

$$\sum_{i=1}^{n}u_i(g_i^*)=1 \tag{6}$$

and

$$u_i(g_{i^*})=0 \quad \text{for all } i. \tag{7}$$

The Eqs. (6) and (7) are then used in the normalization of the utility functions. The first of these equations is used in the normalization of the maximum values of each criterion, indicating that the sum of the maximum values of each criterion is equal to a unit. Eq. (7) meanwhile, attributes the value zero to the initial value of each criterion. In this way, the utility functions which will be determined for each criterion present an initial value equal to zero and the maximum value and the maximum value to be determined through linear programming.

The UTA method uses linear programming to determine the values of the variables, which determine the form of the utility functions of each criterion. In order to perform the implementation of the UTA method it becomes necessary to make a prior multicriteria evaluation. To do this, the matrix of the performances of the alternatives in relation to the criteria, such as the weak ordering R defined in A or A', where A' is a subset of the representative alternatives present in A. For each pair $(a,b)\in A'$, the decision agent expresses a preference or global indifference.

For each alternative of A', the utility function calculated $U'[g(a)]$ differs from the true $U[g(a)]$ by an error $\sigma(\alpha)$

$$U'[g(a)]=\sum_{i=1}^{n}u_i[g_i(a)]+\sigma(a) \quad \text{for all } a\in A'. \tag{8}$$

Considering the relations of preference (3) and relations of indifference (4), as well as the utility function (8), we have

$$U'[g(a)]-U'[g(b)]\geqslant\delta \;\Leftrightarrow\; \text{if the decision agent indicates } aPb, \tag{9}$$

$$U'[g(a)]-U'[g(b)]=0 \;\Leftrightarrow\; \text{if the decision agent indicates } aIb \tag{10}$$

with $\delta>0$ being a real small number used to meaningfully separate two classes of the weak ordering R. The authors of the UTA method suggest, in their original conception of the method, that the value of δ must belong to the interval $[1/10Q, 1/Q]$, with Q being the number of classes of indifference. Assuming the existence of transitivity, the