ARTICLE IN PRESS

Journal of Mathematical Psychology ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

Sequential sampling, magnitude estimation, and the wisdom of crowds

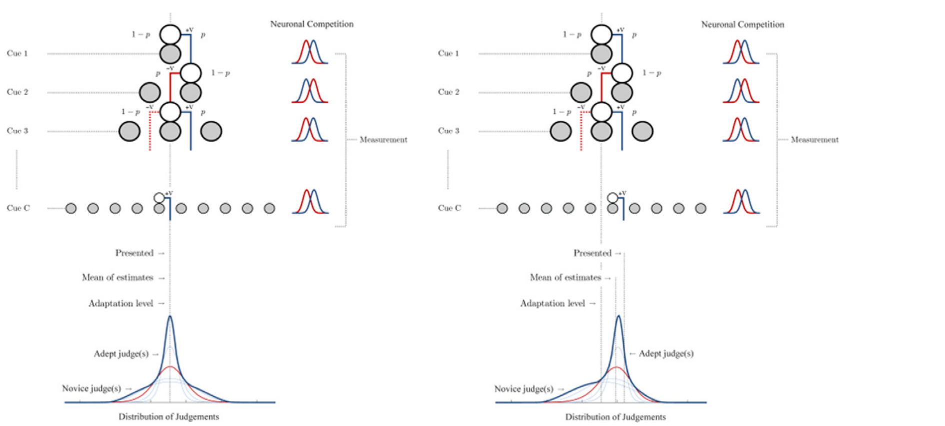
Ulrik W. Nash

University of Southern Denmark, Denmark

HIGHLIGHTS

- We present a neuronal model of probabilistic magnitude estimation.
- We predict the wisdom of crowds is one psychophysical effect in an entire system.
- We conduct an experiment on magnitude estimation and find support for all elements.
- We confirm a procedure for correcting errors in the wisdom of crowds.
- An old conjecture by Sir Francis Galton is settled.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 27 February 2016
Received in revised form
8 December 2016
Available online xxxx

Keywords:
Individual heterogeneity
Sequential sampling
Magnitude estimation
Judgment distributions
Wisdom of crowds
Psychophysics

ABSTRACT

Sir Francis Galton (Galton, 1907) conjectured the psychological process of magnitude estimation caused the curious distribution of judgments he observed at Plymouth in 1906. However, after he published Vox Populi, researchers narrowed their attention to the first moment of judgment distributions and its often remarkable alignment with the truth, while it became customary to explain this wisdom of crowds effect using ideas of statistics more than psychology, and without considering possible interactions with other distribution moments. Recently, however, an exploration of the cognitive foundation of judgment distributions was published (Nash, 2014). The study not only formalized a possible link between signal detection, evidence accumulation, and the shape of judgment distributions, but also in so doing, conjectured that magnitude estimation by independent individuals causes a systematic error in the wisdom of crowds indicated by judgment distribution skewness. The present study reports findings from an experiment on magnitude estimation and supports these predictions. The study moreover demonstrates that systematic errors by groups of people can be corrected using information about the judgment distribution these people together form, before errors might cause damage to decision making. In concluding, we revisit Galton's data from the West of England Fat Stock and Poultry Exhibition in light of what we have discovered.

E-mail address: uwn@sam.sdu.dk.

http://dx.doi.org/10.1016/j.jmp.2017.01.001

Please cite this article in press as: Nash, U. W., Sequential sampling, magnitude estimation, and the wisdom of crowds. Journal of Mathematical Psychology (2017), http://dx.doi.org/10.1016/j.jmp.2017.01.001

1. Introduction

As individuals, our judgments of magnitude are often wrong in the particular, but the mean of guesses by many individuals about something (Galton, 1907b), or even the average of many judgments by one individual about something (Vul & Pashler, 2008), is often remarkably accurate and precise for reasons of probability. Particular judgments are subject to error, but when errors scatter in equal proportion around the truth, the mean is an accurate measurement of things in the world around us. In fact, when every error of underestimation has an equivalent counterpart error of overestimation, the mean judgment is valid and reliable. This phenomenon has been called many things, from Vox Populi (Galton, 1907b) to Rational Expectations (Muth, 1961), to the Many Wrongs Principle (Simons, 2004). Most recently it became known to the general public as Wisdom of Crowds (Surowiecki, 2004).

How the brain harnesses laws of probability to facilitate the wisdom of crowds remains unclear, although we have long suspected the brain itself is probabilistic (Brunswik, 1943; Laplace, 1812), not least because we observe it generating various estimates of the same presented stimulus (Faisal, Selen, & Wolpert, 2008; Luce & Mo, 1965; Stocker & Simoncelli, 2006). We know more about how social mechanisms undermine the mean by turning independent judgments dependent (Lorenz, Rauhut, Schweitzer, & Helbing, 2011; Muchnik, Aral, & Taylor, 2013), but when it comes to explaining independent judgments, we hit an obstacle. Our primary models describe the result of thinking without reference to the cognitive mechanisms that generate these outcomes (Griffiths, Chater, Norris, & Pouget, 2012; Hoffman, 1960). Without an explicit link to the cognitive processes that generate independent judgments, we cannot move beyond statistics to explain collective errors that occur even before crowds are swayed by social forces.

It was recently argued (Nash, 2014) that crowds of independent people make errors of judgment, which are signaled by skewness in the judgment distributions they together form. The argument went beyond the macroscopic level of statistics by offering explanations relating to psychophysical effects at the mesoscopic level, and evidence accumulation following signal detection at the microscopic level of the brain. These explanations were harvested from an augmented version of the Quincunx, the statistical device Sir Francis Galton built in 1873 to demonstrate the Central Limit Theorem (Galton, 1894). From assumptions about the environment and the cognitive system, the AQ emerges as an elegant model of norm-based coding (Kayaert, Biederman, Op De Beeck, & Vogels, 2005; Leopold, Bondar, & Giese, 2006; Loffler, Yourganov, Wilkinson, & Wilson, 2005; Rhodes et al., 2005), signal detection (Britten, Shadlen, Newsome, & Movshon, 1992; Newsome, Britten, & Anthony Movshon, 1989), and evidence accumulation (Latimer, Yates, Meister, Huk, & Pillow, 2015; Shadlen & Newsome, 2001; Yang & Shadlen, 2007), and becomes a probabilistic computer of judgments.

Galton plays an important role in this research article. Besides inventing the original Quincunx, it was Galton who wrote the seminal paper on the wisdom of crowds (Galton, 1907b) and speculated that psychophysicists held the key to explaining his observations. Galton was intrigued by the curious distribution of magnitude estimates he uncovered at Plymouth and speculated about the mental methods that caused it. However, Galton's idea that judgment distributions convey information about cognitive processes has received little attention since, although an early exception was Brunswik's (1956) independent work on the cognitive continuum and his examination of error distributions produced by intuition versus analysis. One reason why few have studied judgment distributions to develop theories about cognition could be the success of paramorphic methods (Hoffman, 1960), or equivalently, what Marr (1982) referred to as studies of the cognitive system at the computational level. Researchers since Galton have developed highly accurate predictions about magnitude estimation, without needing to model how the brain generates fine-grained measurements about the world around it. In particular, regression and Bayesian methods have been successful in this regard.

Had competitors at the West of England Fat Stock and Poultry Exhibition been required to discriminate between the weight of two oxen, as opposed to guessing the precise weight of one, then any question Galton might have posed about cognitive mechanisms would almost certainly have been answered sooner. Indeed, contemporary scientists are relatively knowledgeable about the mechanism used by cognitive systems to discriminate between two magnitudes.

Unlike contemporary studies of precise magnitude estimation, contemporary studies of magnitude discrimination are commonly carried out at what Marr (1982) called the algorithmic level. In particular, sequential sampling models have been argued to capture the essence of an important subset of human cognitive mechanisms to provide accurate predictions about another significant distribution in cognitive psychology, namely the distribution of time taken by individuals to choose between possible responses.

A connection between the cognitive processes of magnitude estimation and discrimination may exist, but probing the connection is not our purpose here. Rather, we aim to suggest that sequential sampling and the wisdom of crowds are linked through magnitude estimation, and along the way, explain why current sequential sampling models of magnitude discrimination cannot readily predict that link. We begin by clarifying what sequential sampling models are, compare the most important of these, and explain their confinement to coarse-grained problems of binary choice. We subsequently introduce the AQ model in detail and highlight why it, on the other hand, can readily be applied to the fine-grained problem of estimation. Having done that, we present predictions by the AQ and report findings from an experiment on magnitude estimation that provides good support. Most importantly, the study demonstrates that systematic errors by groups of people can be corrected using information about the judgment distribution these people together form, before errors might cause damage to decision making. In concluding, we revisit Galton's data from the West of England Fat Stock and Poultry Exhibition in light of what we have discovered.

2. Sequential sampling and the problem of discrimination

When applied to questions of perception, sequential sampling models make fundamental assumptions about the environment on the one hand, and the cognitive system on the other. About the former, the environment is assumed to signal its state, while about the latter, cognitive systems are assumed to sample information sequentially from signals to generate evidence about the environment, which the system accumulates to reduce surprise quickly. As pointed out by Forstmann, Ratcliff, and Wagenmakers (2016), sequential sampling is not simply governed by the availability of signals but is an unavoidable consequence of the cognitive system's inability to process all available information immediately. In other words, sequential sampling is thought to be a defining characteristic of imperfect cognitive systems.

Another premise relating to the limitation of cognitive systems concerns the accuracy of evidence these systems generate from signals. Somewhere in the process, there are sources of error relating to Thurston's (1927) idea of discriminal dispersion, according to which the effect of signals on the cognitive system is probabilistic. The mathematical representation of errors by