International Communications in Heat and Mass Transfer 84 (2017) 27–34

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Effects of severe blockage on heat transfer from a sphere in power-law fluids

T. Vidya Sagar, S.A. Patel [1], A.K. Gupta, R.P. Chhabra *

Department of Chemical Engineering, Indian Institute of Technology, Kanpur 208016, India

ARTICLE INFO

Available online xxxx

Keywords:
Wall effects
Nusselt number
Poiseuille flow
Power-law fluid

ABSTRACT

The momentum and thermal energy equations describing the forced convection heat transfer from a heated sphere settling at the axis of a long cylindrical tube filled with a power-law fluid have been solved numerically. The extensive new results reported herein encompass wide ranges of conditions as: Reynolds number, $1 \le Re \le 100$; Prandtl number, $5 \le Pr \le 100$, power-law index, $0.2 \le n \le 2$ and blockage ratio, $0.5 \le \lambda \le 0.95$. The range of values of the power-law index (n) used here include both the shear-thinning ($n < 1$) and shear-thickening ($n > 1$) fluid behaviours. The overall heat transfer is strongly modulated by Re, n and λ depending upon whether the recirculation region is formed in the rear of the sphere and/or on the proximity of the tube wall. Furthermore, the results reported herein elucidate the effect of the type of thermal boundary condition (isothermal or isoflux) on the surface of the sphere as well as that of the velocity profile (uniform or fully developed Poiseuille profile) in the tube. Overall, the average Nusselt number bears a positive dependence on the Reynolds and Prandtl numbers and blockage ratio. The shear-thinning behaviour ($n < 1$) augments heat transfer over and above the corresponding Newtonian value whereas shear-thickening behaviour ($n > 1$) adversely influences it. The present numerical results (~4000 data) have been consolidated by incorporating the blockage factor into an existing expression valid for $\lambda = 0$ for Newtonian fluids.

1. Introduction

Owing to its fundamental significance, much research effort has been expended in studying the settling of a sphere in power-law fluids, for it constitutes a classical problem in the domain of fluid mechanics. Also, a falling sphere in stationary and moving fluids in cylindrical tubes denotes an idealization of several industrially important applications such as the falling ball viscometer, thermal treatment of food stuffs in tubes, and in modelling slurry pipelines transporting coarse particles in non-Newtonian carriers including the removal of debris in drilling operations [1–3]. While in the falling ball viscometry, the terminal settling velocity of a sphere is measured in a stationary medium, in case of slurry pipelines and thermal treatment of foodstuffs, a particle is exposed to the fully developed velocity distribution which may further get modified depending upon the concentration of particles. In any case, it is the relative velocity between the sphere and the fluid which determines the momentum and heat transfer characteristics. Consequently, over the years, a wealth of information has accrued about various aspects of this configuration like drag, wake and heat transfer characteristics from an unconfined sphere and this body of information has been reviewed in some recent studies, e.g., see Ref. [2]. Broadly, the drag is increased above its Newtonian value accompanied by a concomitant augmentation in heat transfer coefficient (Nusselt number) in shear-thinning fluids ($n < 1$) [2,4]. However, the influence of power-law index (n) on sphere drag is more prominent in the viscosity dominated low Reynolds number regime whereas the varying levels of enhancement in heat transfer are obtained only at moderate Peclet numbers. Combined together, the reliable numerical predictions of drag and Nusselt number are now available up to $Re = 100$ and $Pe = 10^4$. Suffice it to add here that the drag results are consistent with the available experimental results [2,5]. The next generation of studies in this field has dealt with the effect of symmetric and asymmetric confinement by considering the sedimentation of a sphere at the axis [6–11] and off-axis [12,13] locations in cylindrical tubes filled with power-law fluids. In this case additional effects are encountered depending upon the velocity profile in the tube, i.e., whether the sphere settles in a stationary fluid or in the Poiseuille flow conditions. The presence of the confining walls alters the flow field in the close proximity of the sphere due to the upward motion of the fluid displaced by the settling sphere. This sharpens the velocity and temperature gradients on the surface of the sphere thereby augmenting the hydrodynamic drag and the Nusselt number with reference to the corresponding values in unconfined fluids under otherwise identical conditions. The low Reynolds number drag results

* Corresponding author.
E-mail address: chhabra@iitk.ac.in (R.P. Chhabra).
[1] Present address: Department of Chemical, Biological and Pharmaceutical Engineering, New Jersey Institute of Technology, Newark, NJ 07102, USA.

http://dx.doi.org/10.1016/j.icheatmasstransfer.2017.03.005

Nomenclature

c	thermal heat capacity of fluid, J kg^{-1} K^{-1}
d	sphere diameter, m
D	tube diameter, m
h	local heat transfer coefficient, W m^{-2} K^{-1}
k	thermal conductivity of fluid, W m^{-1} K^{-1}
L_d	downstream domain, m
L_u	upstream domain, m
m	power-law consistency index, Pa s^n
n	power-law index, dimensionless
$\boldsymbol{n}_s$	unit normal vector to the surface of sphere, dimensionless
Nu_{avg}	average Nusselt number ($=hd/k$), dimensionless
Nu_{cond}	conduction limit of the average Nusselt number, dimensionless
Nu_θ	local Nusselt number ($=-\partial\xi/\partial n_s$ for CWT; $=1/\xi$ for CHF), dimensionless
Pe	Peclet number ($=\mathrm{Re}\cdot\mathrm{Pr}$), dimensionless
Pr	Prandtl number ($=\frac{c}{k}m(\frac{V_0}{d})^{n-1}$), dimensionless
q_0	surface heat flux, W m^{-2}
Re	Reynolds number ($=\rho d^n V_0^{2-n}/m$), dimensionless
r	radial coordinate, m
T	fluid temperature, K
T_C	temperature of the inlet fluid, K
T_H	temperature on the surface of the sphere, K
V_0	average velocity, m s^{-1}
z	axial coordinate, m

Greek letters

λ	blockage ratio ($=d/D$), dimensionless
ρ	fluid density, kg m^{-3}
θ	angular position on the sphere surface, degree
ξ	temperature of fluid ($=(T-T_C)/(T_H-T_C)$ for CWT, $=(T-T_C)/q_0 d/k$ for CHF), dimensionless

Acronyms

CWT	constant wall temperature
CHF	constant heat flux

for a confined sphere are consistent with the experimental results [2,9], but the two values begin to deviate increasingly with the increasing values of the Reynolds number and/or decreasing value of the power-law index [7,9]. As of now, these results are limited to $\lambda \leq 0.5$ and $Re \leq 100$ [6–11]. The enhancement in heat transfer is ascribed to the thinner thermal boundary layers in power-law fluids ($n < 1$) than that in Newtonian fluids [14]. This body of knowledge is neither as extensive nor as coherent as that for Newtonian fluids [2,15], albeit much of the work relates to the case of a sphere settling at the axis of a cylindrical tube and deals with wall effects on drag behaviour. Intuitively, it appears that as the value of λ is increased beyond $\lambda > 0.5$, the velocity and temperature fields around the sphere are further modified owing to the proximity of the wall. This effect is less severe for $\lambda < 0.5$, for the wall is still sufficiently far away vis-à-vis the thickness of the thermal boundary layers formed on the sphere. It is likely that for high values of λ such as 0.8 or 0.9, due to the fluid acceleration in the annulus formed by the tube wall and the sphere surface leading to relatively large values of the local Reynolds number (compared to its nominal value), further sharpening of the velocity and temperature gradients may occur. This will lead to stronger interactions between the tube wall and the thin boundary layers formed on the sphere. Under some conditions, recirculating regions are also formed along the wall of the tube. This study endeavours to elucidate the role of severe blockage ($\lambda > 0.5$) on heat transfer from a sphere in power-law fluids thereby extending the range of currently available results especially in terms of the blockage ratio.

In particular, the momentum and energy equations have been solved numerically here over the following ranges of conditions of the pertinent governing parameters: Reynolds number ($1 \leq Re \leq 100$); Prandtl number ($5 \leq Pr \leq 100$); power-law index ($0.2 \leq n \leq 2$) and the blockage ratio ($0.5 \leq \lambda \leq 0.95$). The present predictions of drag are validated using the corresponding results for Newtonian and power-law media up to $\lambda \sim 0.5$–0.6 available in the literature while no prior results (numerical or experimental) are available on Nusselt number, even in Newtonian fluids.

2. Problem definition

Since the problem formulation, simplifying assumptions and the governing equations along with the boundary conditions are detailed in previous studies [6,8–11,16], these are not repeated here. Fig. 1 shows the two cases of a sphere (of diameter d) falling with an average velocity V_0 in a quiescent fluid in a tube of diameter D, Fig. 1(a) and that of the sphere exposed to the fully developed profile (Poiseuille flow, Fig. 1(b)). Naturally, the two cases differ only in terms of the boundary conditions. No-slip condition ($V_r = 0, V_z = 0$) is used both on the surface of the sphere and on the walls. For temperature, the wall is assumed to be adiabatic and the condition on the surface of sphere is that of constant temperature $\xi = 1$ for the isothermal and $-\partial\xi/\partial\boldsymbol{n}_s = 1$ for the constant heat flux condition. The other boundary conditions are shown in Fig. 1. Dimensional considerations of the governing equations and boundary conditions in both cases suggest the heat transfer characteristics (isotherm contours, local and average Nusselt number) to be functions of the following four dimensionless parameters:

$$Reynolds\ number:\ Re = \frac{\rho {V_0}^{2-n} d^n}{m}$$

$$Prandtl\ \ number:\ Pr = \frac{c}{k} m \left(\frac{V_0}{d}\right)^{n-1}$$

$$Power-law\ index: n$$

$$Blockage\ ratio: \lambda = \frac{d}{D}.$$

The local Nusselt number,Nu_θ, is simply given by $-\partial\xi/\partial\boldsymbol{n}_s$ for the isothermal case, and by $1/\xi$ for the constant heat flux case. Here, $\boldsymbol{n}_s$ is the unit vector normal to the surface of the sphere. Such local values of the Nusselt number can be integrated over the surface of the sphere to obtain the mean Nusselt number, Nu_{avg}. Evidently, for a given thermal boundary condition and the type of velocity profile in the tube, the average Nusselt number is expected to be a function of Re, Pr, n and λ. This work endeavours to explore and develop this functional relationship.

3. Numerical methodology and choice of parameters

The non-linear momentum and energy equations have been solved using the finite element based method COMSOL Multiphysics® (version 4.3a), as detailed in our previous studies [7–9,16,17]. Suffice it to say here that very fine triangular mesh of element size within a range of 7.75×10^{-4}–5.42×10^{-2} was employed close to the surface of the sphere (up to a radial distance of $d/2$) and the rest of the region was meshed using quadrilateral elements of non-uniform size to economize on the required computational resources. A simulation was deemed to