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A mixed convection flow and heat transfer of pseudo-plastic power law nanofluids past a stretching vertical plate

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ABSTRACT

A mixed convection flow and heat transfer of pseudo-plastic power law nanofluid past a stretching vertical plate is investigated. Three types of nano-particles, such as copper (Cu), aluminum oxide (Al_2O_3) and titanium oxide (TiO_2), are considered. The generalized Fourier law proposed by Zheng for varying thermal conductivity of nanofluids, which is dependent on the power law of velocity gradient as well as the nanoparticles property, is taken into account. Dual solutions are obtained by Bvp4c with Matlab. The stability of the solution also is discussed by introducing the unsteady governing equations. Furthermore, a new interesting phenomenon is found: the local Nusselt number do not maintain the similar characteristics of Newtonian fluid near the point where the velocity ratio is equal to 0.5.

1. Introduction

In recent years, many investigations have been made to explore the nanofluid technology in the field of enhanced heat transfer and new generation cooling technology, which overcomes the limitation of many conventional heat transfer fluids with low thermal conductivity, such as water, oil, and ethylene glycol mixture. Furthermore, due to the tiny size of nanoelements and the low volume fraction of nanoelements required for conductivity enhancement, nanofluids are also very stable and have no additional problems [1], such as sedimentation, erosion, additional pressure drop and non-Newtonian behavior. Numerous methods have been taken to develop advanced heat transfer fluids with substantially higher conductivity by suspending nano(usually less than 100 nm)/micro or larger-sized particle materials in liquids [2]. The comprehensive references and the broad range of current and future applications on nanofluids can be found in the recent book [3] and in the review papers by Buongiorno [4],Wang and Mujumdar [5], Kakac and Pramuanjaroenkij [6], Wong and Leon [7].

Nanosfluid has been used to improve heat transfer for some convection problems due to its wide applications in electronics cooling, heat exchangers, and double pane windows. Furthermore, mixed convection is preferred and then many numerical studies about heat and mass transfer of nanofluids in enclosures have been studied [8–11]. Some similarity solutions or analytical solutions also have been done. For example, Nield and Kuznetsov [12,13] studied the free convection of viscous and incompressible nanofluid past a vertical plate embedded in porous medium or not. Loganathan et al. [14] discussed unsteady natural convection flow of nanofluids past an infinite vertical plate with considering the radiation.Ahmad et al. [15] extended Blasius problem and Sakiadis problem to the case of nanofluids. Norfifah Bachok et al. [16] made another extension and extended the Blasius and Sakiadis problems in nanofluids by considering a uniform free streem parallel to a fixed or moving flat plate. Ahmad and Pop [17] investigated steady mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids. One of interesting problems, the existence of dual solutions, also have been proposed, which bring more insight on engineering applications. For example, Subhashini et al. [18–20] pointed out that the upper branch solutions are most physically relevant solution whereas the lower branch solutions seem to deprive physical significance or may have realistic meaning in different situations. Furthermore, the stability of the numerical solutions for the mixed convection also has been analyzed. Since Mahmood and Merkin [21] done the classical work and investigated the mixed convection on a vertical circular cylinder, Merkin [22], Merkin and Pop [23] also discussed the dual solutions occurring in mixed convection in a porous medium by perturbation method and analyzed the stability of the upper and lower branch of the solutions. Similarly, some interesting works also have been done [24,25].

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In contrary, the number of studies on natural and mixed convection of nanofluids, where Non-Newtonian power law fluids is considered as the base fluid, is very small. Lin and Zheng et al. [26–28] studied the Marangoni convection of power law nanofluids, where they considered the thermal conductivity to be dependent on velocity or temperature gradient. Some other mass and heat transfer models about power law nanofluids [29–32] also have been proposed and solved numerically or analytically.

The aim of this paper is to investigate the mixed convection mass and heat transfer of power law nanofluids past a stretching vertical plate. Here the CMC-Water (0.0–0.4%) is considered as a pseudo-plastic power law fluid [26–28] and regarded as the base fluid. Three types of nanoparticles are considered: copper (Cu), aluminum oxide (Al_2O_3) and titanium oxide (TiO_2). According to the experimental studies, the thermophysical properties of CMC-Water(<0.4%) are similar to water [33,34]. Their related properties are given in Tables 1 and 2. The generalized Fourier law proposed by Zheng [26–28,35,36] for varying thermal conductivity of nanofluids is taken into account. The similar equations are solved numerically with Matlab. Effects of different parameters on velocity and temperature are discussed in detail.

2. Governing equations

Consider the steady two-dimensional mixed convection flow and heat transfer of power law nanofluid over a vertical porous stretching plate. The base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. Here we assume that x-axis is along the vertical surface and y-axis is normal to the plate. u and v are the velocity components in the x and y directions, respectively. As shown in Fig. 1, the free stream velocity and the stretching velocity are assumed to be $u_e = a$ and $u_w = b$. The temperature T_w at the wall is a constant, T_∞ is the temperature of the fluid far away from the plate. The suction/injection velocity along the plate is $v_w = -f_w \frac{1}{n+1}(v_f U^{2n-1} x^{-n})^{\frac{1}{n+1}}$, where $U = a + b$ is the composite velocity.

Under these assumptions, the governing equations about the mixed convective flow and heat transfer of power-law nanofluid can be written as follow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}}\frac{\partial}{\partial y}\left(\mu_{nf}\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + g\frac{(\rho\beta)_{nf}}{\rho_{nf}}(T - T_\infty), \tag{2.2}$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa_{nf}\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial T}{\partial y}\right), \tag{2.3}$$

where $\rho_{nf},\ k_{nf},\ \mu_{nf},\ (\rho c_p)_{nf},\ \beta_{nf}$ are the density of the nanofluid, the thermal conductivity of the nanlfluid, the modified viscosity of the nanofluid, the heat capacitance of the nanofluid and the coefficient of the thermal expansion, respectively. $\mu = \mu_{nf}|\frac{\partial u}{\partial y}|^{n-1}$ is the effective viscosity of the nanofluid. $K = \alpha_{nf}|\frac{\partial u}{\partial y}|^{n-1}$ is the thermal diffusivity proposed by Zheng et al. [26–28,35,36], where $\alpha_{nf} = \frac{\kappa_{nf}}{(\rho c_p)_{nf}}$ is the modified thermal diffusivity of the nanofluid. The expression of above nanofluid parameters are given as follow:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,\ (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s,$$
$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s,\ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$
$$\frac{\kappa_{nf}}{\kappa_f} = \frac{(\kappa_s + 2\kappa_f) - 2\phi(\kappa_f - \kappa_s)}{(\kappa_s + 2\kappa_f) + \phi(\kappa_f - \kappa_s)}, \tag{2.4}$$

where $\rho_f,\ \rho_s$ are the reference density of the fluid fraction and the solid fraction, and $k_f,\ k_s$ are the thermal conductivity of the fluid fraction and solid fraction, respectively. ϕ is the solid volume fraction parameter of the nanofluid, μ_f is the viscosity of the fluid fraction. Here the modified viscosity μ_{nf} of the nanofluid can be approximated as viscosity of the base fluid μ_f containing dilute

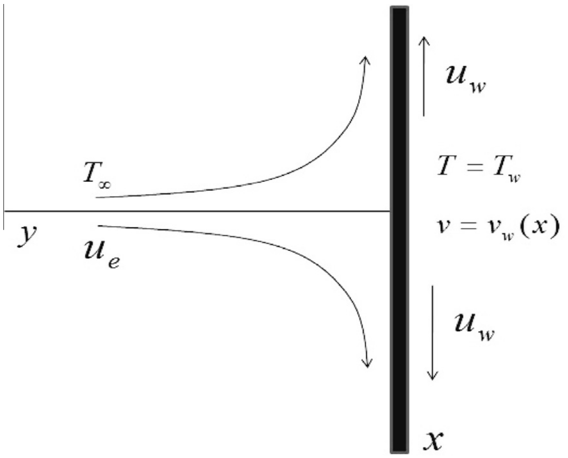

Fig. 1. The model of heat and mass transfer of mixed convection.

Table 1
Thermophysical properties of the base fluid and nanoparticles.

Property	CMC-water (0.0–0.4%)	Copper (Cu)	Aluminum Oxide (Al_2O_3)	Titanium Oxide (TiO_2)
c_p(J/kg K)	4179	385	765	686.2
ρ(kg/m^3)	997.1	8933	3970	4250
κ(W/mK)	0.613	400	40	8.9538
$\beta \times 10^{-5}(1/K)$	21	1.67	0.85	0.9

Table 2
Dynamical properties of the carboxy methyl cellulose water.

	0.0	0.1	0.2	0.3	0.4
The power law index	1.0	0.91	0.85	0.81	0.76
K(Ns^n/m^2)	8.550×0^{-4}	6.319×10^{-3}	1.754×10^{-2}	3.136×10^{-2}	7.853×10^{-2}