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Correlations for the symmetric converging flow and heat transfer between two nearly parallel stationary disks similar to a solar updraft power plant collector

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ABSTRACT

By focusing on the derivation of the corresponding friction factor and Nusselt number for constant temperature absorber, this work considers the internal symmetric sink flow and heat transfer between two nearly parallel disks stationary similar to the fluid motion in the collector of a Solar Updraft Power Plant. For that, a numerical steady state turbulence model for buoyant incompressible fluids exploiting the variation of the duct cross-section was developed. Friction factor and Nusselt number correlations are suggested for intermediate range of Reynolds numbers, where thermo-hydrodynamic aspects are important for Solar Updraft Power Plant performance assessments.

1. Introduction

Symmetric flow and heat transfer between two nearly parallel stationary disks is a particularly interesting phenomenon which has found many industrial and scientific applications such as in Solar Updraft Power Plant (SUPP) collectors, centrifugal compressors, air bearings, radial diffusers, vertical take-off aircraft, air cushion vehicles, rotating heat exchangers, air bearings, geothermal reservoirs, etc. (Murphy et al., 1978; Kettleborough, 2002). Thanks possibly to an interest in the fluid dynamics of radial diffusers, the scientific community has paid much more attention to radially diverging and decelerating flow, e.g. (Woolard, 1957; McGinn, 1955; Jackson and Symmons, 1965a,b; Licht and Fuller, 1954; Livesey, 1960; Moller, 1961; Andreyeva, 1968; Savage, 1964; Ishizawa, 1964; Wilson, 1972; Boyack and Rice, 1970; Detry et al., 2009; Chen et al., 2009; Chatterjee, 2008). The same tendency could be noticed for heat transfer studies regarding rotatory disks (Chen et al., 2009; Djaoui et al., 2001; de Beer et al., 2014; Fitzgerald and Garimella, 1998; Poncet and Serre, 2009; Poncet and Schiestel, 2007; Petukhov, 1970; Garimella, 2000; Harmand et al., 2013; Hsieh and Lin, 2005; Iacovides and Theofanopoulos, 1991; Owen, 1971; Owen and Rogers, 1989; Pellé and Harmand, 2007; Shen et al., 2014; Eshita, 2014).

For the converging laminar flow between two disks, McGinn (1955) originally developed theoretical pressure distribution correlations bringing forward the argument that the pressure variation is partially due to the inertial contribution and partially due to viscous dissipation. In one such study (Garcia, 1969), the experimental analysis on the unsteady air flow between two disks of returned velocity fluctuations measurements indicates the manifestation of an intrinsically unstable flow. Lee and Lin (1985) derived simplified dimensionless expressions for the pressure gradient by linearizing of the Navier-Stokes equation and replacing radial velocity with mean radial velocity. Livesey (1960) and Savage (1964) solved these equations taking into account the inertial term and using integral approach and the assumption of parabolic velocity profile. Savage (1964) presented a series solution by perturbing the creeping-flow solution. Complimentarily, Vatistas (1988) employed the method of separation variables to solve a model incorporating the inertial term and linearizing the inertial term in the momentum equation by taking radial velocity from the continuity equation averaged over the gap, returning static pressure distribution, radial component of the velocity and, friction coefficient correlations. Likewise, Vatistas et al. (1995) revealed a numerical solution for the description of the static pressure distribution and the radial velocity. Additionally, the power series approach was employed to solve the same problem (Zitouni and Vatistas, 1997). Singh et al. (1999) developed an experimental and numerical analysis involving the measurement of velocity field

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with laser Doppler anemometry and the mean governing equations using $k-\varepsilon$ model. The numerically predicted velocity and pressure values returned good agreements with experimental data.

Murphy et al. (1978) studied numerically the steady, laminar, incompressible axisymmetric flow converging radially between two stationary disks. They identified three distinct flow regions comprising a region of strong viscous effects (dimensionless radii[1] $\gg 1$), a region of equivalent viscous and inertial effects (dimensionless radii $\sim$1) and a region of strong inertial effects (dimensionless radii $\ll 1$). By means of some acceleration parameters at the extremes of dimensionless radii, they discuss the applicability of laminar results including works of Patel and Head (1968) and Kays (1966). Advanced studies of this team included experiments (Murphy et al., 1983) and indicated through flow and pressure distribution visualization that the flow regime remains laminar for local Reynolds numbers between 210 and 20,700. The flow subjected to acceleration due to lateral convergence exhibits relaminarization features. Their succeeding work (Chambers et al., 1983) substantiated their previous work (Murphy et al., 1983). Although all flow characteristics varied with acceleration, the bursting[2] frequency was the most affected one. Conducting more intricate experiments regarding internal turbulent flows and turbulent boundary layers, McEligot and Eckelmann (2006) concluded that strong pressure gradients reduce the transport of momentum in the outer part of the viscous layer. Their most recent works (McEligot et al., 2009) released that the unsteady behaviors of the dominant contribution to entropy generation were recognized to be much more sensitive to distance from the wall than to streamwise pressure gradient.

The paper (Jawarneh, 2013) researched two disks flow with inlet swirl both experimentally and numerically regarding some geometry parameters and Reynolds number achieving good agreement with experimental data. Ultimately, approximate analytical solutions for converging flow in between two parallel non rotating disks were derived by the work (Wee and Gorin, 2015) and compared with experimental results and other works (Livesey, 1960; Savage, 1964; Vatistas, 1988; Kwok et al., 1972).

As opposed to these findings, the authors assert that the velocity profile does not preserve its parabolic shape as the fluid flow in the direction of the center. Concerning heat transfer analyses of rotating converging flow, the paper (Pellé and Harmand, 2009) investigates the local heat transfers on a rotor surface in the air-gap of a discoidal rotor–stator system, in which an air jet comes through the stator and impinges the rotor disclosing critical radii over the rotor surface and corresponding correlations. The work of dos Santos Bernardes (2003) makes use of the $k-\varepsilon$ model to investigate numerically the steady, turbulent, incompressible flow and heat transfer converging radially between two stationary disks, which can be considered an extension of the work of Murphy et al. (1978).

The work confirmed the fluid dynamic flow behavior reveals correlations for the Nusselt number relate to two regions, namely not and fully thermally developed regions. Solar Updraft systems are solar power generating plants which convert solar radiation gathered in the collector into kinetic energy by means of buoyancy in the chimney. Turbines positioned at the chimney base are used to produce electricity. The literature is extensive, and that referred to here is by no means exhaustive. The works (dos Santos Bernardes, 2010; Zhou and Xu, 2016; Zhou et al., 2010) review some of the outstanding issues at that time. The problem to be addressed here refers to the heat transfer process between the collector roof and the flowing air. 1-D Simulations conducted by two relevant researchers, namely Bernardes (dos Santos Bernardes, 2004; dos Santos Bernardes et al., 2003) and Pretorius (Pretorius, 2004; Pretorius and Kröger, 2008; Pretorius and Kröger, 2006a,b; Pretorius and Kröger, 2007; Pretorius and Kröger, 2009; Pretorius and Kröger, 2008; Fluri et al., 2009) introduced respectively classical flat plate and experimental correlations systematically discussed in the paper (Aurélio dos Santos Bernardes et al., 2009). The work (dos Santos Bernardes, 2011) also introduces some plausible heat transfer coefficients applicable to a SUPP collector. However, an ultimate solution for the thermo-hydrodynamic process taking place in a SUPP collector could not be found in the previous literature. It is noteworthy to mention that most of the described hydrodynamic solutions assume that the air flows radially into the disc gap neglecting that the flow finally have a vertical path. At this point the flow has to turn through 90 deg and significant exit effects are expected. These solutions are thus not the most appropriate for flows in SUPP collectors.

Thus, the problem to be addressed here is the axisymmetric flow (no azimuthal variation) between two finite stationary disks, similar to the fluid motion in the collector of a SUPP, concentrating on radially converging turbulent flow development and heat transfer.

Also, at variance with previous works and aiming to preserve flow similarities, the present study includes in the outlined geometry the region in the immediate vicinity of the channel exit, i.e. ‘curve’, and a cylindrical duct denoting the SUPP chimney. Flow and heat transfer features of these entities are beyond the scope of this work. Likewise, the influence of the roof design – occasionally regarded as a constant height along the collector (dos Santos Bernardes, 2004; dos Santos Bernardes et al., 2003; Schlaich, 1995) or as constant cross area (Kröger and Blaine, 1999; Pretorius and Kroger, 2006) – in the thermo-fluid dynamics processes in collector is also estimated. Furthermore, we confine our attention to mass flow and Reynolds number ranges usually disclosed by relevant researches, e.g. (Zhou et al., 2010; dos Santos Bernardes et al., 2003; Pretorius and Kröger, 2006a; Krätzig, 2013). Deeper analysis regarding turbulence structures, relaminarization and, flow accelerations phenomena as developed in the works (Murphy et al., 1978, 1983; Chambers et al., 1983; McEligot and Eckelmann, 2006; McEligot et al., 2009) are not examined here.

2. Physical and mathematical model

Fig. 1 outlines the physical model and flow in question, consisting in the collector, curve and chimney. It consists of a gap between two stationary circular disks, with the origin for the transverse coordinate y located at center of the chimney. Here, the variables r, y, v, u denote the radial coordinate, the vertical coordinate, and the vertical and radial components of velocity, respectively.

The flow cross section is kept constant in the chimney. The variable b in Eq. (1), namely roof shape coefficient, was introduced by the work (Kettleborough, 2002) to explore its influence on the flow behavior. Thus, the flow cross section varies in the collector from a constant height ($b = 0$) to a constant cross section ($b = 1$). The curve plays a link function between the collector and the chimney. At this point, the flow cross section at the curve varies linearly joining collector outlet and chimney inlet. Accordingly, the actual roof height increases toward the center of the collector is described by Eq. (1).

$$h = H_2\left(\frac{r_2}{r}\right)^b \begin{cases} b = 0: & \text{constant height} \\ b = 1: & \text{constant flow section} \end{cases} \tag{1}$$

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[1] Dimensionless radii = radial coordinate/(half the disc spacing × Re_0 (overall Reynolds number)); overall Reynolds number = volumetric flow rate/(4 × π × kinematic viscosity × half the disc spacing).

[2] Bursting means here the quasi-deterministic occurrence of large-scale organized structures. This phenomenon is supposed to play a dominant role in the development of turbulent boundary layers.