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Exact approximations for skin friction coefficient and convective heat transfer coefficient for a class of power law fluids flow over a semi-infinite plate: Results from similarity solutions

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A B S T R A C T

In the present work, it is aimed to derive exact formulations for skin friction coefficient (C_f) and convective heat transfer coefficient (h) for power law fluids with $0.8 < n < 1.2$ flow over a semi-infinite plate and with the constant wall temperature boundary condition. Similarity analysis was employed to solve the problem for 410,000 cases with different power law indexes (n) and Prandtl numbers ($0.1 < Pr < 1000$). Finally, based on the acquired data from similarity solutions, exact approximations (with R-Square ~ 1) are proposed for C_f and also h. These exact formulations can be considered as replaces for the former ones in the literature for Newtonian fluids ($n = 1$).

1. Introduction

Since many fluids follow non-Newtonian behavior [1–3], it is worth to conduct researches which aim to extend studies on this class of fluids. Among many works in this field, some of them can be found in [4–10]. These works are mainly referred to similarity solution in which the method has been widely employed in the literature for solving boundary layer problems. It must be also cited that similarity procedure has proved itself efficient even for providing solutions for verity of complex fluid flows [11–14].

The goal of the present research is specialized to study a class of power law fluids ($0.8 < n < 1.2$) flow over a semi-infinite plate with constant wall temperature boundary condition in a detailed way.

Here, we mention that among the many models for non-Newtonian fluids, power law model is applicable mainly because of its simplicity, and it only approximately predicts the treatment of a real non-Newtonian fluid. As an example, if the power law index (n) is less than one, the power law predicts that the effective viscosity would decrease with the increase of shear rate indefinitely, which requires a fluid with infinite viscosity at rest and zero viscosity as the shear rate gets close to infinity. But a real fluid has both a minimum and a maximum effective viscosity that rely on the physical chemistry at the molecular level. Therefore, the power law stands as only a suitable description of fluid behavior across the range of shear rates to which the coefficients could be fitted to the experiments [3].

Beside the limitations of power law model, there are many fluids which follow a power law behavior with power law indexes close to unity. This class of fluids can simply include variety of suspensions (Ex. nanofluids (2016) [15]). Furthermore, the value of power law index equal to unity stands as an exception in the nature; so, many assumed Newtonian fluids (especially in higher temperatures) may have power law treatments with values of "n" close to 1 (see (2016) [16,17]). Therefore having specified information for this class of fluids can subsequently benefit us in exact engineering evaluations of this class of fluids.

Two main engineering factors (skin friction coefficient and convective heat transfer) have been targeted in this work. These factors are evaluated and formulated as functions of Reynolds number, Prandtl number and power law index. These formulations are already derived for Newtonian fluids ($n = 1$) in the same conditions; but the need for exact results in engineering problems led us to extend these formulations to include much wider class of fluids.

Using power law model, two classes of Non-Newtonian fluids can be relatively (but not completely) studied (pseudo-plastic and dilatant which is less common). This model simply defines the shear stress as:

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Nomenclature

Symbol	Description (unit)		
Re	Reynolds number	μ	dynamic viscosity (kg/m.s)
T_w	surface temperature (K)	ρ	density (kg/m^3)
υ	kinematic viscosity (m^2/.s)	n	power law index
Pr	Prandtl number	α	diffusivity factor
h	convective heat transfer coefficient (W/m^2.K)	C_f	skin friction coefficient

$$\tau = K\left|\left(\frac{\partial u}{\partial y}\right)\right|^{n-1}\left(\frac{\partial u}{\partial y}\right) \tag{1}$$

which K is the consistency factor and "n" is the power law index. Which n < 1 covers a range of pseudo-plastic fluids and n > 1 includes a class of dilatant fluids [3]. Substituting this model for predicting shear stress leads us to the following governing momentum equations [5]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\upsilon\left(\frac{\partial u}{\partial y}\right)^{n}\right)=\frac{\partial}{\partial y}\left(\upsilon\left|\left(\frac{\partial u}{\partial y}\right)\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{3}$$

Note that y-momentum equation can be simply ignored with respect to the order of magnitude.

Moreover using $\alpha_t=\alpha\left|\frac{\partial u}{\partial y}\right|^{n-1}$ as the thermal diffusivity for power law fluids [5], energy equation can be written as:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\left|\left(\frac{\partial u}{\partial y}\right)\right|^{n-1}\frac{\partial T}{\partial y}\right) \tag{4}$$

Eqs. (2)–(4) are transformed into two ODEs by defining a similarity parameter. Among the classic methodologies of solving these ODEs (some of them are employed in [18]), Shooting Technique procedure with a simple forward discrete scheme was employed in the present research. After validating the code for the classic Blasius case, it has been applied for numerous values of 0.8 < n < 1.2 and Pr > 0.1 (410,000 cases of different values of "n" and "Pr" were solved in this research).

It is well-known that for Newtonian fluids (n = 1), $C_{fx}=\frac{0.664}{\sqrt{\mathrm{Re}_x}}$ and $h_x=0.332\frac{k}{x}\mathrm{Re}_x^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}$; but there are not yet the same formulations (approximations) for power law fluids. So, power law forms of these equations are introduced in here for 0.8 < n < 1.2.

Applying the transformed ODEs for 410,000 cases of 0.8 < n < 1.2 and 0.1 < Pr < 1000 and using Least Square Regression Techniques, skin friction coefficient and convective heat transfer are accurately formulated (R-Square ∼ 1). As previously mentioned, these new formulations can be considered as replaces for evaluation of these two important factors in the literature.

2. Deriving the transformed ODEs for power law fluids

Using order of magnitude technique, the governing PDEs for power law fluids flow over a semi-infinite plate can be obtained as Eqs. (2) and (3):

In which $\upsilon=\frac{K}{\rho}$.

As the classic Blasius solution, here we assume that the x-component of velocity is self-similar. So, this component of velocity can be written as:

$$\frac{u}{U_0}=F(\eta) \tag{5}$$

By assuming similarity parameter as $\eta=ayx^{b}$ (a > 0), stream function can be obtained as follows:

$$\begin{aligned}\psi&=\int u dy\\ \psi&=\int U_0F(\eta)dy\\ \psi&=\int\frac{U_0F(\eta)}{ax^{b}}d\eta=\frac{U_0}{ax^{b}}f(\eta):f=f(\eta)\end{aligned} \tag{6}$$

By definition of stream function, the continuity equation (Eq. (2)) will be automatically satisfied. Furthermore, the velocity components can be obtained as:

$$u=\frac{\partial\psi}{\partial y}=U_0f' \tag{7}$$

$$v=-\frac{\partial\psi}{\partial x}=-\frac{1}{a}x^{-b}f(-bx^{-1}U_0)-U_0x^{-1}ybf' \tag{8}$$

The derivatives of x-component velocity are then obtained as:

$$\frac{\partial u}{\partial x}=U_0f''(aybx^{b-1}) \tag{9}$$

$$\frac{\partial u}{\partial y}=U_0f''ax^{b} \tag{10}$$

$$\frac{\partial^2 u}{\partial y^2}=U_0a^2x^{2b}f''' \tag{11}$$

Substituting Eqs. (7)–(11) into x-momentum equation (Eq. (3)) results in:

$$ff''\left(\frac{bU_0^2}{x}\right)=\upsilon na^2U_0x^{2b}f'''(U_0f''ax^{b})^{n-1} \tag{12}$$

Eq. (11) can be re-written as:

$$-ff''=f'''f''^{(n-1)}\left(-\frac{\upsilon na^{n+1}U_0^{n-2}x^{b(n+1)+1}}{b}\right) \tag{13}$$

The similarity solution exists if only $b(n+1)+1=0$.

So, $b=-\frac{1}{n+1}$. By this definition of b, the multiplying factor in the right-hand side of Eq. (13) would be a constant. For simplification, this constant can be assumed as to be unit in value. Therefore:

$$-\frac{\upsilon na^{n+1}U_0^{n-2}}{b}=1 \tag{14}$$

Note that in the left hand side of Eq. (14), all the parameters except b, is positive; so the right hand side of Eq. (14) must be either positive. Moreover, selecting the right hand side of Eq. (14) equal to 1, is just a simplifying assumption. Therefore, mathematically, it may take any value arbitrarily.

From Eq. (14), the constant factor of "a" can be obtained as:

$$a=\left(\frac{U_0^{2-n}}{n(n+1)\upsilon}\right)^{\frac{1}{n+1}} \tag{15}$$

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