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Uncertainty propagation in FE modelling of a fire resistance test using fractional factorial design based model reduction and deterministic sampling

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ABSTRACT

In this paper, fractional factorial design (FFD) and deterministic sampling (DS) are applied to a finite element (FE) model of a fire resistance test of a loaded steel beam, to investigate how uncertainties are propagated through the FE model. The sought quantity was the time when the deflection of the beam exceeded 225 mm. The FFD method was used as a model reduction technique which reduced the number of uncertain parameters from 5 to 3. The DS method was compared to a reference Monte Carlo (MC) method of 1000 simulations from all 5 uncertain parameters, which was the minimum number of simulations in order for the statistical moments to converge. The combined FFD and DS method successfully computed the propagation of the mean and standard deviation in the model, compared to the MC method. Given the uncertainties in the FE model, the fractional factorial design reduced the number of simulations required in the DS method by 82%. The combined method of FFD and DS reduced the number of required simulations by 96% compared to the MC method. The DS method did not capture the tails of the probability distribution and is therefore not a suitable candidate for probabilistic evaluation of the time of failure at the edges of the domain of possible failure times. Future research could very well be on improving the tails in DS. However, the DS method provides a conservative 95% coverage interval of 6 min for the time to failure of the steel beam.

1. Introduction

Due to the lack of knowledge of the precise details of a system, seemingly random behavior can be governed by deterministic, though non-linear models. Thus, in such systems, small variations may have a significant impact on the outcome. Moreover, modelling is often performed in stages by different numerical or analytical tools where the uncertainties may propagate through the system without much consideration or control [1]. It is therefore usually a very difficult task to objectively establish the confidence levels in numerical predictions. Uncertainty Quantification (UQ) is the science of quantitative characterization and reduction of uncertainties in numerical studies and real world experimentation. UQ aims at determining how likely certain outcomes are if some aspects of the system are unknown.

There is clearly a growing interest in the use of probabilistic methods in structural fire engineering. This is driven by a desire to be able to adequately quantify the level of safety of a structural design in probabilistic terms, along the same basis as the principles of reliability which underpin the structural Eurocodes [2].

Examples in fire engineering which account for uncertainties mostly rely on brute force Monte Carlo simulation to evaluate the uncertainties in models of, e.g. reinforced concrete columns [3], pre-stressed concrete beams [4], reinforced concrete slabs [5], or steel beams [6]. Methods which have been developed for reliability analysis have been incorporated within the OpenSEES software to add probabilistic hazard analysis and reliability calculation of structures for fire [7]. Furthermore, development of fragility curves of steel buildings for fire loading [8], and the application of the PEER PBEE framework to structures in fire [9,10] have been investigated.

However, random sampling can be both computationally expensive and time consuming. Other work on the reliability of fire life safety and fire suppression systems proposes the use of response surface modelling and linear regression techniques to identify critical variables for reliability estimation and iterative algorithms to reduce the number of calculations required [11,12]. These techniques lead to less computationally expensive analyses than traditional Monte Carlo techniques.

As an alternative to random sampling techniques, in this paper we explore the application of deterministic sampling to understand the impact of uncertainties on structural fire engineering problems. Deterministic sampling originates from the field of signal processing

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Nomenclature

h	convective fire heat transfer coefficient (W/m^2K)
h_c	ambient convetive heat transfer coefficient (W/m^2K)
ε	relative emissivity
E	Young's modulus (GPa)
Y	yield stress (MPa)
F	load (kN)
L	length of steel beam (m)
L_S	span between supports (m)
L_F	distance from support to applied load (m)
$T_{ad.}$	adiabatic boundary condition (K/m)
T_f	standard time-temperature curve, EN 1363-1 (°C)
y	time when deflection criteria is exceeded (min)

and the most widely used methods are extensions of the Kalman filter which is based on the work of Wiener [13]. Consider a random variable that propagates through a function. The statistical moments of that function do not actually depend on the number of samples, but rather that the samples fulfill the statistical moments of the distributions they come from. This is the principle that deterministic sampling works under. Thus, deterministic sampling is the art of choosing samples that fulfills known statistical moments.

Factorial design provides a methodology for designing experiments such that main effects and interactions can be determined [14,15]. If some of the factors have a greater effect on the response of a system compared to other factors, this method can be used as model reduction technique i.e. reducing the number of uncertain parameters. Further, a fraction of the experiment design can be used instead of the full factorial design. This is called fractional factorial design.

In this paper, fractional factorial design and deterministic sampling are applied to an FE model of a fire resistance test of a loaded steel beam. The aim is to investigate how the uncertainties propagate through the FE model. The results from the factorial design and the deterministic sampling will be compared to Monte Carlo analysis of the same model where no uncertain parameters are excluded.

2. Theory

2.1. Factorial design

Many experiments and numerical simulations involve the study of several factors. A factorial design means that in each replication all possible combinations of the levels of the factors are investigated. The effect of a factor is defined as the change in the response produced by a change in the level of a factor. This is referred to as the main effect of a factor. Consider a simple example involving only two factors, A and B. They each have two levels, a high level and a low level. Denote them '+' and '-' respectively. Thus there are four possible responses. They are summarized in Table 1.

The main effect of factor A is the difference between the average response at the high level of A and the average response at the low level of A i.e.

$$\mathrm{A} = \frac{52+40}{2} - \frac{30+20}{2} = 21$$

Similarly the main effect of B is

$$\mathrm{B} = \frac{52+30}{2} - \frac{40+20}{2} = 11$$

Thus the main effect of A is larger than the main effect of B in this case. This calculation can be written as an algebraic system. First define the design matrix $\boldsymbol{D}$,

$$\boldsymbol{D} = \begin{bmatrix} +1 & +1 \\ -1 & -1 \\ +1 & -1 \\ -1 & +1 \end{bmatrix} \tag{1}$$

where the first column contains the possible levels of factor A and the second column contains the possible levels of factor B, for the four different combinations. Second, define the response matrix

$$\boldsymbol{R} = \begin{bmatrix} 52 \\ 20 \\ 40 \\ 30 \end{bmatrix} \tag{2}$$

which contains all four possible responses of the factor combinations. Lastly, define the effect matrix

$$\boldsymbol{E} = \frac{2}{n}\boldsymbol{D}^T\boldsymbol{R} = \frac{1}{2}\begin{bmatrix} +1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}\begin{bmatrix} 52 \\ 20 \\ 40 \\ 30 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 52-20+40-30 \\ 52-20-40+30 \end{bmatrix} = \begin{bmatrix} 21 \\ 11 \end{bmatrix} \tag{3}$$

where n is the number of responses. Matrices (1) - (3) defines the frame work of factorial design [14,15]. Sometimes, the difference in response between the levels of one factor is not the same at all levels of the other factors. This is called an interaction. Consider another set of responses for factors A and B.

The four possible interactions, AB, between A and B can be introduced as an extension of the design matrix (1)

$$\boldsymbol{D} = \begin{bmatrix} +1 & +1 & +1 \\ -1 & -1 & +1 \\ +1 & -1 & -1 \\ -1 & +1 & -1 \end{bmatrix} \tag{4}$$

Where the third interaction column is the product of the two levels in column one and two. The response matrix using the values in Table 2 will be

$$\boldsymbol{R} = \begin{bmatrix} 12 \\ 20 \\ 50 \\ 40 \end{bmatrix} \tag{5}$$

The effect matrix will then read

$$\boldsymbol{E} = \frac{2}{n}\boldsymbol{D}^T\boldsymbol{R} = \frac{1}{2}\begin{bmatrix} +1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 \\ +1 & +1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 12 \\ 20 \\ 50 \\ 40 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 12-20+50-40 \\ 12-20-50+40 \\ 12+20-50-40 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -29 \end{bmatrix} \tag{6}$$

Where the interaction AB is −29. If the response matrix corresponding to Table 1 is used instead in (6), the interaction AB would be only 1. In a two factor factorial design there can only be an interaction between A and B, which is referred to as a second order interaction. If there were for example 3 factors, there would also be a third order interaction ABC, in addition to the second order interactions, AB, AC and BC.

Table 1
A two factorial design with four possible responses.

	A^+	A^-
B^+	52	30
B^-	40	20