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Threading a multifractal social psychology through within-organism coordination to within-group interactions: A tale of coordination in three acts

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ABSTRACT

This manuscript attempts an informal, relatively concept-heavy/mathematics-lean presentation to all experts on group processes about how many group processes might unfold upon a generic sort of scaffolding called "multifractal structure." Explaining group processes poses complementary challenges of explaining similarity among agents belonging to a group and, also, explaining frustrating dissimilarities when agents pull apart and begin to wander from the fold, showing deep multi-scaled texture (e.g., groups containing subgroups, agents containing subagents). Whereas time-varying, multi-scaled texture hampers many linear models, multifractality does what so few other formalisms can: it allows predicting similarities and dissimilarities from nonlinear interactions across scale. Empirical estimates of the multifractal spectrum offer continuously-varying but compact logical support for portraying both the qualitative similarities and the more frustrating qualitative dissimilarities. This story begins at one level to meet the organism at an intuitively behavioral scale, zooms in to a within-organism view, and zooms out to an across-organism view. At each view, resonance of multifractal modeling with the multi-scale structure of group processes reveals new insights into how group behaviors support perception, action, and cognition. This tale of social coordination told in three separate acts has a moral: Multifractality may be a ready tool for wider social-psychological application.

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1. Introduction

This work offers a brief overview of a complexity-themed view on psychological processes. To this view, all psychology is effectively a psychology of groups. There are no atomisms completely apart from one another, at least not in the activities and events that span the bandwidth typically assumed for behavior of living things (e.g., milliseconds to years; [1]). In a progression anticipated by Mandelbrot's [2] and Richardson's [3] insights, gazing into the private details of a single organism's behavior only reveals more texture and more interactivity amongst constituent parts, and zooming out to take the view of an organism in its social interactions might only show similar texture and interactivity over longer scales. To this zoom-capable view, the notion of scale-invariance takes on a thrilling possibility that the variability at one scale might resemble variability at another and, more to the point, that interactions within the organism may follow the same geom-

etry of interactions across the multiple organisms. In this sense, there is no sort of psychology that is not social.

We may be quite selective about the form worth expecting to characterize these interactions appearing at each scale. The sketch need not sound too permissive or fanciful, but a newcomer to this literature might appreciate the direct warning that specificity is going to come in the form of a word often classified as "jargon" by those who do not know it yet. That word is "multifractal," and it is important enough to warrant first a more casual description for newcomers (Section 1.1) and, only after that, a formal definition to meet those readers seeking more mathematical rigor (Section 1.2). Before reading either, we might reflect together on the urgent and non-multifractal needs that multifractal math might address: Specifically, we might expect multifractal structure specifically because of a growing consensus that the mind—in all of its social, cognitive, perceptual, and emotional facets—thrives on nonlinear interactions across scales [4–6]. The most important reason to move from that consensus to worrying over a definition for "multifractality" is that models of nonlinear interactions across scales yield multifractal structure, suggesting that multifractal modeling should provide a direct window into specifically

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those nonlinear interactions across scales that propel the mind and its goal-directed expressions through behavior. Hence, no matter the scale on which we rest our zoom-in-zoom-out-able view, we should see the mind charting a behavioral course that has roots at many more than one scale—some of which may be the scales we have just, for now, dropped from our refocusable lens. And I propose here that a really good lens for this view is multifractal analysis and/or the capacity to articulate multifractal structure.

1.1. A casual definition of multifractal structure

An excellent question at this juncture is "What is multifractal structure?" Readers looking for a more formal answer should skip to the next paragraph and past the more casual answer that immediately follows. The rest of this paragraph offers a relatively casual answer first for the newcomer: Multifractal structure is the tendency for distributions (of whatever behavior/event you are interested in) to be so heterogeneous as to need a description more than just in terms of means and standard deviations—which two statistics make up most of the currency of statistical language in behavioral science. Besides varying in their average location (mean) or their spread (standard deviation), distributions might have nontrivial (i.e., fascinating) internal structure that warrant further specification. Lewis Fry Richardson noticed this need for internal structure when he recognized patches and clumps in the dispersion of fluid molecules [3], and he settled on a powerful way to quantify this clumpiness in terms of the number of neighboring particles at progressively larger scales. Namely, Richardson found that, for any individual particle, the number of neighboring particles would drop off nonlinearly but also very slowly. As we move our view of a cloud of particles from single particle to a larger span, the number of new neighbors revealed at each larger span will dwindle, but it will not vanish abruptly to zero because clouds of particles are poorly contained and sprawling entities. The mathematical expression that Richardson chose and that Mandelbrot [2] promoted for posterity was the power-law, a mathematical relationship between size (or number or any measurement of magnitude) and scale that specifies slow decay (or growth, in the reverse direction). This slow decay extends across many scales and persists such that, over many scales, the decay follows precisely the same form. The decay depends entirely on an exponent that relates size (e.g., number of neighbors in Richardson's example) to scale: that is, the power-law for neighboring particles would be "Number of Neighbors is proportional to Scale^m," and power-law structure entails that m specifies the decay throughout all scales. This single power law is called "fractal structure" because it corresponds to a kind of diffusion called "fractional Brownian motion" [7]. However, the reason for multiplying the sorts of fractality is that, according to the example of the particle cloud metaphor, we had only needed the power-law because we were counting the neighbors for a single particle. If we begin the same procedure from a different particle, and if the cloud of particle is actually heterogeneous and clumpy, then starting out with different particles will yield different power-law relationships even in the same cloud. That is, the exponent m described above may not describe how neighbor counts decay for all particles. We may need to keep track of exponent m_1 and exponent m_2 for Particles 1 and Particle 2, but the possibility is that keeping track of the diversity of exponents m might be the best way to describe the heterogeneous cloud under observation. It may help reading of what follows after the next paragraph to know that one sort of exponent m that will appear below is called the "Hurst exponent" and that the more locally-definable exponents m_1 and m_2 are called "Holder exponents."

1.2. A more formal definition of multifractal structure

More formally, and against the risk of proliferating another description with my own accidents of interpretation, we may consider the words of Bogachev and Bunde [8] recently published in Physical Review E for a more defensible definition of multifractality in terms of nonlinear long-term memory:

Long-term memory can be either (i) linear, (ii) nonlinear, or (iii) both linear and nonlinear. In the first case, which is often referred to as "monofractal," the (linear) autocorrelation function $C_x(s)$ of the data decays with time s by a power law, $C_x(s) \sim s^{-\gamma}$, $0 < \gamma < 1$, and the exponent γ fully describes the correlations within the record. … In the second case, where the record is "multifractal," the linear autocorrelation $C_x(s)$ vanishes for $s > 0$ and nonlinear (multifractal) correlations, which cannot be described by a single exponent, characterize the record.

We can also speak formally about the relationship between the linear autocorrelation function decay exponent γ relates to "fractal dimension" and also to Hurst exponents and to the long-memory of an ARFIMA process or in power-law decaying Fourier coefficients—all of which are equivalent or bijective for linear systems [9,10]. However, these elegant formal relationships have diminishing returns particularly when, as in the present case of pondering nonlinear interactions across scales, the heterogeneity of the analyzed systems/data arise from nontrivial relationships between relatively local and relatively global scales of behavior. That is to say, the formalisms and equivalences we might draw are elegantly simple for homogeneous systems and for linear correlations, but the formalized relationships become systematically less useful as the system under observation has more heterogeneity and more interesting intrinsic structure [11,12].

1.3. An apt test case for studying systems with nontrivial intrinsic structure: psychology

Whether one reads the casual introduction to multifractality or the more formalized definition borrowed from the physics literature, the issue of multifractality centers on how to describe systems with nontrivially nonlinear intrinsic structure. Psychology is an excellent test case in which cognitive science points quite plainly to the complicated intrinsic structure in the form of so-called internal models. I wish to put psychology and its appeal to internal models on the chopping block here—a chopping block that two very important cyberneticians had a crucially eloquent hand in specifying for posterity. Psychologists will put their money on internal models largely because the intrinsic structure of a behaving organism—let alone a group of those organisms—have such convoluted intrinsic that it puts a severe limit on what science can observe, e.g., with the naked eye.

In particular, those two cyberneticians were eager that the limits of observation should not color or cripple rigorous theory. No matter what scale an observer focuses on, there is always the risk of complacently assuming that we have somehow settled on the only relevant scale of activity. The warnings from Ashby [13], a psychiatrist better known for his cybernetics work, are particularly instructive: "as soon as some of a system's variables become unobservable, the 'system' represented by the remainder may develop remarkable, even miraculous, properties" (p. 113). It is important to note that, long before this special issue aimed to bring the study of complex systems to bear on psychological outcomes of group dynamics, Ashby himself was very explicit about coming to complex systems with psychology in mind. He knew that this "developing" of properties was a psychological artifact of inevitable limits of observation, and he knew as well that this risk of inventing miraculous properties was perhaps greatest when we