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Bidirectional and multidirectional associative memories as models in linkage analysis in data analytics: Conceptual and algorithmic developments

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A R T I C L E I N F O

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A B S T R A C T

Associative and bidirectional associative memories are examples of associative structures studied intensively in the literature and exhibiting a large volume of applied studies. The underlying idea is to reveal and describe linkages among data and express them in a form of an associative mapping. Such mappings are constructed in a way so that the recall processes (both one-directional and bidirectional) lead to the recalled items characterized by a minimal recall error. Associative memories, morphological memories, and fuzzy associative memories have been studied in numerous areas yielding efficient applications to image recall and enhancements and fuzzy controllers (which can be regarded as one-directional associative memories). In this study, we revisit and augment the concept of associative memories by offering some new conceptual design insights where the corresponding mappings are realized on a basis of a related collection of landmarks (prototypes) over which an associative mapping becomes spanned. In light of the bidirectional character of mappings, we develop an augmentation of the existing fuzzy clustering (Fuzzy *C*-Means) in the form of a so-called collaborative fuzzy clustering. Here an interaction in the construction of prototypes is optimized so that the bidirectional recall error can be minimized. Further conceptual architectural augmentations are discussed including a relational description of associative memories and linkage analysis accomplished in the presence of explanatory spaces. We generalize the mapping into its granular version in which numeric prototypes formed through the clustering process are made granular so that the quality of the recall can be quantified. Several scenarios of allocation of information granularity aimed at the optimization of the characteristics of recalled results (information granules) quantified in terms of coverage and specificity are proposed. Illustrative examples are presented as well.

1. Introduction

Associative memories have been intensively studied for a number of decades being driven by several fundamental and applied research. The fundamentals origin from the interest in intriguing ways to understand how memories organize, store and retrieve data; there are some biologically-inclined investigations playing a visible role [7,17,18,20]. The crux of the recall in associative memories is to produce (recall) an item associated with some available chunk of data being incomplete or noisy.

The term “association” plays a pivotal role in human endeavors and goes back to the ideas of Hebbian (correlation) learning and so-called correlation associative memories. The studies on associative memories reported in the existing literature revolve along concepts such as stability, recall properties (error), capacity, robustness, to name a few of commonly explored ideas. Central to associative memories are the issues of knowledge representation, design practices, and performance analysis and optimization.

The advances in fuzzy sets and neurocomputing have resulted in the emergence of so-called fuzzy associative memories; one can refer here to [3,14,28] that highlight the main representative architectures of fuzzy associative memories. The pertinent design practices are discussed in [8,9,10,12,24,25,26,29,31] addressing issues of stability and the quality of recall as well neural network-based realizations. The development of associative memories comes with a variety of frameworks including logic-oriented [21] and quantum logic environment [23,30]. Cellular-based automata approach was studied in [16] and graph-based approaches [4], hierarchical structures [11] were also investigated. The wealth of applications is profound including tackling parameter estimation and control problems [13,15,27], decision-support systems [1,22] and formal concept analysis [2]. Also significant are several embodiments of mod-

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els and data-linkages come from the context of textual semantics [5,6].

Associative memories come with their own topologies, properties development strategies and ways of carrying out performance evaluation. When talking about topologies, we distinguish between single layer and multilayer memories. The recall process can be done in two directions; in this situation we are referring to bidirectional memories. Memories can concern two-valued data (typically assuming values −1 and 1) or continuous ones assuming values in [0,1] or the space of real numbers. The performance of the memory is quantified in terms of a certain performance index; in case of a bidirectional recall, the performance index expressed as a sum of distances between the results of recall obtained for some given object located in $\boldsymbol{X}$ and the original item and those distances formed when the recall is carried out for objects given in $\boldsymbol{Y}$. The capacity of the memory is quantified as the number of pairs of items that can be stored in the memory and recalled without any error.

There are three outstanding features of the proposed approach, which distinguish it from the existing constructs of associative memories:

First, we develop a general direction-free associative memory so that a recall mechanism is constructed in a general way by making sure that the recall is being realized efficiently in both directions to assure the quality of the bi-directional recall,

Second, the proposed approach focuses on the formation of a bidirectional associative mapping, which is spanned over a collection of landmarks (prototypes). This aspect is crucial as the construct focuses on the essential components of the mapping not being impacted by some marginal and irrelevant linkages between the spaces in which associative mappings are formed,

Third, as we discuss the design details of the memory, it will be demonstrated that its development is realized by not sharing data present for individual spaces. Along this line discussed are selected ways of involvement of additional linkage spaces.

The paper is structured in a way that reflects the main concepts and ensuing algorithmic design practices. In Section 2, we start with brief highlights of the essence of the associative mapping by characterizing it as an optimization problem. We advocate its structure by showing that the mapping spans over a collection of representatives of data for which associations are to be built. We identify some conceptual advantages as well as deliver an efficient way of building the memories. Section 3 is focused on the algorithm behind the buildup of the structural core of memories, which realizes a collaborative version of fuzzy clustering followed by the optimization problem arising in conjunction of the optimal recall (Section 4). Section 5 is focused on the architectural enhancements of linkage analysis including relational linkage analysis, multisource associations, and associations with auxiliary (explanatory) spaces. A granular augmentation of the memory (resulting in a so-called granular associative memories) is presented in Section 6; we show how to form granular prototypes. It is also shown how having this granular augmentation of the memory becomes useful in quantifying the quality of the memories. Experimental studies are reported in Section 7. Conclusions are included in Section 8.

2. Bidirectional associative memories: building mappings spanned over data representatives

In a nutshell, a bidirectional associative memory can be regarded as a pair of mappings f: $\boldsymbol{X} \rightarrow \boldsymbol{Y}$ and g: $\boldsymbol{Y} \rightarrow$ X where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are subsets of associated items (data) positioned in the multi-dimensional spaces of real numbers $\boldsymbol{R}^n$ and $\boldsymbol{R}^m$, respectively. The mappings are formed in such a way they produce a minimal recall error for any finite number of pairs of items stored in the memory. More specifically, f and g are constructed in such a manner that

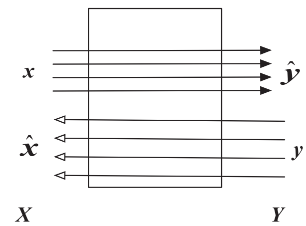

Fig. 1. Bidirectional associative memory with the optimization requirements on achieving closeness between original items in the spaces $\boldsymbol{X}$ and $\boldsymbol{Y}$.

for all pairs of stored items (associations) $(\boldsymbol{x}_k, \boldsymbol{y}_k)$ $\mathbf{x}_k \in \mathbf{R}^n$, $\mathbf{y}_k \in \mathbf{R}^m$, $k=1,2, \ldots, N$, the following recall conditions are (approximately) satisfied, $f(x_k) \approx y_k$ and $g(y_k) \approx x_k$, $k=1, 2, \ldots, N$. Evidently, it is unlikely to anticipate that the associative recall could be ideal. Instead, some recall error occurs hence the ultimate objective of the overall design process is to develop the structure of the mappings (f and g) as well as optimize their parameters in such a way that a certain performance index V (quite commonly expressed as the sum of the squared errors where $||\ .\ ||^2$ denotes a weighted Euclidean distance) becomes minimized

$$V = \sum_{k=1}^{N} \left\| \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k \right\|^2 + \sum_{k=1}^{N} \left\| \boldsymbol{y}_k - \hat{\boldsymbol{y}}_k \right\|^2 \quad (1)$$

where $\hat{\mathbf{x}}_k, \hat{\mathbf{y}}_k$ are the results of recall of the associated item (data), namely $f(\mathbf{x}_k) = \hat{\mathbf{y}}_k$ and $g(\mathbf{y}_k) = \hat{\mathbf{x}}_k$. Note that we assess the quality of recall completed in both directions, viz. for the individual items in $\boldsymbol{X}$ and $\boldsymbol{Y}$. The essence of the bidirectional recall is visualized in Fig. 1. It is essential to emphasize that the associative recall has to exhibit optimality when realizing mappings in both directions.

As stressed earlier, the aim of the optimization is to minimize the values of the performance index V by selecting the mappings from some classes of admissible mappings $\boldsymbol{F}$ and $\boldsymbol{G}$ and estimating the values of the parameters of f and g, say $f(\boldsymbol{x},\boldsymbol{a})$ and $g(\boldsymbol{y}, \boldsymbol{b})$. In essence, the optimization problem is formulated as follows

$$(f_{opt},\ g_{opt}, \boldsymbol{a}_{opt}, \boldsymbol{b}_{opt}) = argMax_{f \in \boldsymbol{F}, g \in \boldsymbol{G}; \boldsymbol{a}, \boldsymbol{b}} V \quad (2)$$

In what we present here, constitutes a radical shift from the main lines of study existing in the literature in the following way: we advocate that the essence and generality of associative mappings should not reflect numerous detailed data but rather it has to be built upon a collection of representatives of $\boldsymbol{X}$ and $\boldsymbol{Y}$ by forming a family of prototypes resulting in the sequel in so-called association pairs. By focusing on the buildup of the associative memory on such landmarks, two motivating factors are worth underlining:

(i) we can avoid an immediate and direct reliance on noisy and distorted data. The prototypes are a kind of summarization of the overall data and in this way, they capture the essence of the data and tend to ignore details that quite likely be a result of noise manifestation in the data
(ii) the structure of the mapping dwells upon the prototypes and the choice of the prototypes helps set up a certain tradeoff between detailed recall and noise immunity.

Once the prototypes (representatives) have been formed, the details of the mapping are established in terms of the structure of the associative linkages, especially the parameters of the mappings and the way in which some interpolation/approximation aspects are realized.

From the design perspective, the problem naturally splits into the two fundamental phases:

(i) building representatives (landmarks) of associative mappings and their further refinements (optimization). This includes also a realization of a certain form of the associative mapping