



# Blow-up of a hyperbolic equation of viscoelasticity with supercritical nonlinearities

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## Abstract

We investigate a hyperbolic PDE, modeling wave propagation in viscoelastic media, under the influence of a linear memory term of Boltzmann type, and a nonlinear damping modeling friction, as well as an energy-amplifying *supercritical* nonlinear source:

$$\begin{cases} u_{tt} - k(0)\Delta u - \int_0^\infty k'(s)\Delta u(t-s)ds + |u_t|^{m-1}u_t = |u|^{p-1}u, & \text{in } \Omega \times (0, T), \\ u(x, t) = u_0(x, t), & \text{in } \Omega \times (-\infty, 0], \end{cases}$$

where  $\Omega$  is a bounded domain in  $\mathbb{R}^3$  with a Dirichlet boundary condition. The relaxation kernel  $k$  is monotone decreasing and  $k(\infty) = 1$ . We study blow-up of solutions when the source is stronger than dissipations, i.e.,  $p > \max\{m, \sqrt{k(0)}\}$ , under two different scenarios: first, the total energy is negative, and the second, the total energy is positive with sufficiently large quadratic energy. This manuscript is a follow-up work of the paper [30] in which Hadamard well-posedness of this equation has been established in the finite energy space. The model under consideration features a *supercritical* source and a linear memory that accounts for the full past history as time goes to  $-\infty$ , which is distinct from other relevant models studied in the literature which usually involve subcritical sources and a finite-time memory.

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## 1. Introduction

### 1.1. The model and literature overview

Viscoelastic materials demonstrate properties between those of elastic materials and viscous fluid. In the nineteenth century, Boltzmann [14] realized that the behavior of these materials should be modeled through constitutive relations that involve long but fading memory. In particular, Boltzmann initiated the classical linear theory of viscoelasticity. As a consequence of the widespread use of polymers and other modern materials which exhibit stress relaxation, the theory of viscoelasticity has provided important applications in materials science and engineering. Please see [19] (and references therein) for the fundamental modeling development of linear viscoelasticity. We also refer the reader to the monographs [25,49] for surveys regarding the mathematical aspect of the theory of viscoelasticity. In addition, the literature is quite rich in various results on well-posedness and asymptotic stability of hyperbolic PDEs and conservation laws with *memory* terms of Boltzmann type, see for instance [17,18,20–24,26,29,34,39] and the references therein.

In this manuscript, we investigate the following nonlinear hyperbolic equation of viscoelasticity:

$$\begin{cases} u_{tt} - k(0)\Delta u - \int_0^\infty k'(s)\Delta u(t-s)ds + |u_t|^{m-1}u_t = |u|^{p-1}u, & \text{in } \Omega \times (0, T) \\ u(x, t) = 0, & \text{on } \Gamma \times (-\infty, T) \\ u(x, t) = u_0(x, t), & \text{in } \Omega \times (-\infty, 0], \end{cases} \quad (1.1)$$

where the unknown  $u(x, t)$  is an  $\mathbb{R}$ -valued function defined on  $\Omega \times (-\infty, T)$ , and  $\Omega \subset \mathbb{R}^3$  is a bounded domain (open and connected) with smooth boundary  $\Gamma$ . Our results extend easily to bounded domains in  $\mathbb{R}^n$ , by accounting for the corresponding Sobolev embedding, and accordingly adjusting the conditions imposed on the parameters. The system (1.1) models the wave propagation in viscoelastic material under the influence of frictional type of damping as well as energy-amplifying sources. Here,  $|u_t|^{m-1}u_t$  ( $m \geq 1$ ) represents a nonlinear damping which dissipates energy and drives the system toward stability, while  $|u|^{p-1}u$  ( $1 \leq p < 6$ ) represents a nonlinear source of *supercritical* growth rate which models an external force that amplifies energy and drives the system to possible instability. The memory integral  $\int_0^\infty k'(s)\Delta u(t-s)ds$  of the Boltzmann type quantifies the viscous resistance and provides a weak form of energy dissipation by assuming that the relaxation kernel satisfies:  $k'(s) < 0$  for all  $s > 0$  and  $k(\infty) = 1$ . It also accounts for the full past history as time goes to  $-\infty$ , as opposed to the finite-memory models where the history is taken only over the interval  $[0, t]$ .

Nonlinear wave equations under the influence of damping and sources have been attracting considerable attention in the research field of analysis of nonlinear PDEs. In [28], Georgiev and Todorova considered a nonlinear wave equation with damping and sources:

$$u_{tt} - \Delta u + |u_t|^{m-1}u_t = |u|^{p-1}u, \quad \text{in } \Omega \times (0, T), \quad (1.2)$$

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