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# Data-driven heterogeneity in mathematical learning disabilities based on the triple code model

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ABSTRACT

Many classifications of heterogeneity in mathematical learning disabilities (MLD) have been proposed over the past four decades, however no empirical research has been conducted until recently, and none of the classifications are derived from Triple Code Model (TCM) postulates. The TCM proposes MLD as a heterogeneous disorder, with two distinguishable profiles: a representational subtype and a verbal subtype. A sample of elementary school 3rd to 6th graders was divided into two age cohorts (3rd − 4th grades, and 5th − 6th grades). Using data-driven strategies, based on the cognitive classification variables predicted by the TCM, our sample of children with MLD clustered as expected: a group with representational deficits and a group with number-fact retrieval deficits. In the younger group, a spatial subtype also emerged, while in both cohorts a non-specific cluster was produced whose profile could not be explained by this theoretical approach.

## What this paper adds

Researchers into numerical processing, mathematical cognition and learning disabilities have proposed several classifications for heterogeneity in mathematical learning disabilities (MLD) from different theoretical perspectives over the past four decades. Recently, a few empirical studies have focused on investigating these classifications (Bartelet et al., 2014; Pieters et al., 2015; Skagerlund & Träff, 2016; von Aster 2000), explaining their results according to the main etiological approaches, i.e. number module deficits or access deficit. However, none of these studies addressed Triple Code Model (TCM) postulates regarding heterogeneity in MLD. TCM is one of the most cited theoretical paradigms used to explain number processing. It postulates two subtypes of MLD (Dehaene, Piazza, Pinel & Cohen, 2003): a representational subtype (with domain-specific deficits in processing quantities) and a verbal subtype (with phonological deficits leading to arithmetical impairments). Given the strong support for the relationship between phonological processing and arithmetical learning, it is surprising that no studies have been undertaken addressing the hypothesis of a verbal subtype in MLD. This is the first study focused on testing TCM classification using a data-driven strategy among select groups, and which confirms a representational subtype and a verbal subtype. In the younger group, a spatial subtype emerged as well, while a large proportion of children did not classify as predicted by TCM postulates, indicating other possible etiological explanations or subtypes. This research will help in the design of specific identification and intervention programs for children with MLD, and they should be configured according to these findings. Evaluation and intervention programs should address these possible phenotypes in MLD based on their specific weaknesses and strengths, going beyond the general MLD label.

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## 1. Introduction

A large proportion of children (3–6%; Shalev, Auerbach, Manor & Gros-Tsur, 2000) have difficulties in learning mathematics. The principal deficits presented by children with mathematical learning disabilities (hereafter MLD) or dyscalculia are a high error rate in calculation, using immature calculation strategies, and a difficulty in storing and retrieving numerical facts from long-term memory (Geary, 2010). Given the interconnection between the cognitive systems involved in the development of numerical skills and in mathematical learning, there have been a variety proposals explaining MLD, a learning disorder defined by its heterogeneity (Henik, Rubinstein & Ashkenazi, 2015; Karagiannakis, Baccaglini-Frank & Papadatos, 2014; Kaufmann et al., 2013; Price & Ansari, 2013; Wilson & Dehaene, 2007). Of these existing proposals, none have studied heterogeneity from Triple Code Model (hereafter TCM) postulates (Dehaene & Cohen, 1995; Dehaene et al., 2003), one of the most cited theoretical frameworks when explaining number cognition. Thus, the aim of this study was to search for subtypes in MLD as postulated by the TCM.

In the last four decades many classifications of heterogeneity in MLD have been proposed (Ashcraft, Yamashita & Aram, 1992; Dehaene et al., 2003; Geary, 1993, 2010; Ginsberg, 1977; Kosc, 1974; Wilson and Dehaene, 2007), but it was not until recently that empirical research was conducted to study and demonstrate the possible cognitive profiles within this specific learning disorder (Bartelet, Ansari, Vaessen & Blomert, 2014; Pieters, Roeyers, Rossel, van Waelverde & Desoete, 2015; Skagerlund & Träff, 2016; von Aster, 2000). Previous pieces of research have found children with MLD with specific impairments in the approximate number system (Bartelet et al., 2014; Pieters et al., 2015; Skagerlund & Träff, 2016; von Aster, 2000) or with difficulties in recovery of number facts during calculation. These difficulties have been explained by a deficit in mapping symbols, the so-called “access deficit” (Bartelet et al., 2014; Skagerlund & Träff, 2016; von Aster, 2000), and by difficulties in storing and recovering phonological information from long-term memory (Pieters et al., 2015; von Aster, 2000). Other forms of MLD identified during these empirical studies were due to spatial deficits (Bartelet et al., 2014).

Various causes may underlie the pathological learning of mathematics and result in heterogeneity; that is, there exist different profiles within the disorder (Karagiannakis et al., 2014). This is the first empirical study of heterogeneity in MLD based on the postulates of TCM. Recent neurofunctional research with adults and children supports its findings both in participants with normal brain functioning (Moeller, Willmes & Klein 2015) and in children with MLD (Kucian et al., 2014). According to the Triple Code Model (TCM; Dehaene & Cohen, 1995; Dehaene et al., 2003) adults are able to process quantities in three formats that are interconnected from a neurofunctional point of view: analogue representation, verbal representation and visual-Arabic representation. This allows us to handle the quantities using different notations and to solve calculations quickly and efficiently. In this sense, and according to the principles of the model, current research is highlighting the role of verbal processes, especially phonological processing, in mathematics learning (Durand, Hulme, Larkin & Snowling, 2005; Krajewski & Schneider, 2009; LeFevre et al., 2010; Vokovic & Lesaux, 2013). Furthermore, the influence of phonological processing on storage and retrieval of number facts from long-term memory has been the focus of several studies, reviews and proposals (De Smedt, Taylor, Archibald & Ansari, 2010; Hecht, Torgesen, Wagner & Rashotte, 2001; Mammarella, Caviola, Cornoldi, & Lucangeli, 2013; Simmons & Singleton, 2008), and findings have supported the principles of TCM: number facts are stored in phonological format in long term memory without any semantic processing during recovery.

The TCM postulates two subtypes within the MLD category (Dehaene et al., 2003): a subtype with deficits in processing and representation of quantities, and a verbal subtype. In the case of the representational subtype, we expect to find higher error rates when estimating on the mental number line and in spatial processing. This representational deficit could be demonstrated by greater distance effects than for the group without MLD, as has been shown in previous research (Andersson & Östergen, 2012; Ashkenazi, Mark-Zigdon & Henik, 2009; Mussolin, Mejias & Noël, 2010; Price & Ansari, 2013). On the other hand, regarding the verbal subtype, we hypothesize that these children will present difficulties in retrieval of numerical facts from long-term memory due to phonological deficits (De Smedt et al., 2010; Dehaene et al., 2003; Geary, 1993; Geary, 2010; Hecht et al., 2001; Mammarella et al., 2013; Simmons and Singleton, 2008). This latter group has tended to be neglected by researchers of heterogeneity in MLD, despite the amount of evidence supporting the theory.

Heterogeneity should be conceptualized with caution. Traditionally, subtypes have been conceptualized as all-or-nothing entities, but it should be noted that typically, developing individuals show great within-individual and between-individual variability in their patterns of strengths and weaknesses (Cowan et al., 2011; Dowker, 2015; Jordan et al., 2009).[1] When talking about heterogeneity of a developmental disorder, we are searching for profiles of weaknesses and strengths, rather than pure subtypes. Research focused on the study of heterogeneity in MLD has pursued this goal from theoretical perspectives other than that of TCM, clustering their samples by means of data-driven strategies or cut-off criteria. In this regard, various studies have shown a preference for data-driven strategies when selecting groups with difficulties, as opposed to finding a priori subtypes through cut-off strategies or IQ-discrepancies (Bartelet et al., 2014; Brankaer et al., 2014; Lubke & Muthén, 2005; Wong, Ho & Tang, 2014), as the former avoid the groups being limited a priori. The use of this methodology assumes that subtypes are not closed or pure clusters, but that they behave as dynamic entities whose members are grouped by similarities in their performance and cognitive profile, accepting natural intra-group variability.

In the present research, specific tasks allowing measurement of the processes involved in arithmetic were given to participants with MLD. We expected to find patterns of specific performance within the MLD category according to TCM predictions on heterogeneity (Dehaene et al., 2003): a representational subtype and a verbal subtype. Children in the representational subtype were expected to present deficient number representation and spatial deficits, while children in the verbal subtype were expected to

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