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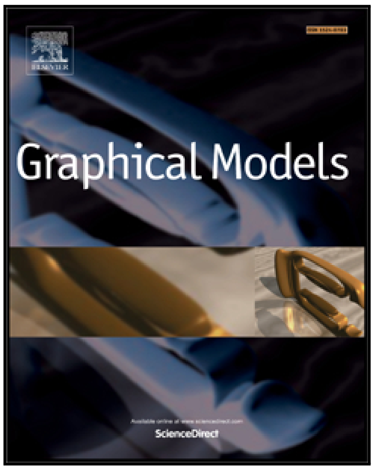

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Computing Medial Axis Transformations of 2D Point Clouds

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Abstract

In this paper, we propose a robust method to compute the medial axis transformation of a 2D point cloud with noise and/or missing data. The basic approach is to first compute the signed distance function of the point cloud by solving the Eikonal equation, and then an approximation of the signed distance function is obtained by sparse optimization technique. The medial axis of the point cloud corresponds to the non-smooth ridge of the distance function which can be extracted by checking the norm of the gradient of the distance function. The medial axis is segmented into branches and a compact spline representation of each branch can be obtained. We perform experiments on various examples and compare our method with state-of-the-art methods. Experimental results demonstrate that our method outperforms previous methods in obtaining accurate and reliable representations for medial axis transformations of noisy 2D point clouds.

Keywords: Medial axis transformation, distance function, sparse optimization, point cloud.

1. Introduction

The concept of medial axis was firstly introduced by Blum in the late 1960s as a shape descriptor in biological shape recognition [1], and later it was generalized to higher dimensions by mathematicians and was called symmetric set or central set [2, 3]. The original idea of medial axis is very simple. Consider starting a grass fire along a curve in the plane, and suppose the fire starts at the same moment. Then the medial axis is the set of locations where the front of the fire meets itself. Geometrically, the medial axis of a planar object is the set of all points having more than one closest point on the boundary of an object, or equivalently, the medial axis is the locus of the centers of circles that are inscribed in the object. The medial axis together with the radius function of the maximally inscribed circles is called the medial axis transformation (MAT for short).

The medial axis representation of an object is a complete shape descriptor of the object, and it has a wide range of applications including image analysis for shape recognition [4], motion planning [5], shape segmentation [6], reverse engineering [7], feature detection [8], shape deformation [9], shape analysis [10], and so on.

So far there have been numerous methods proposed to compute medial axis transformations of 2D and 3D objects, such as topological thinning methods [11, 12], voronoi diagram based methods [13, 14, 15], evolution based methods [16, 17], methods based on distance functions [18, 19, 20, 21, 22, 23, 24, 25, 26], algebraic methods[27, 28], etc. However, how to obtain an accurate and topologically correct medial axis transformation is still a question valuable to be investigated [29]. Even if the boundary representation of the object is exact (e.g. free form shapes [30]), such computation is still difficult. Thus various approaches have been put forward to prune a large portion of extra branches of the medial axis in order to get a stable part of the medial axis, and these approaches include θ-medial axis[31, 32], λ-medial axis[33, 34], scaled medial axis[13, 35], edge-collapsing[15, 36], erosion thickness(ET)[37], etc. In a recent survey paper by Tagliasacchi et al. [29], an overview of start-of-the-art skeletonization methods is summarized. These methods generally compute a major part of the medial axis with the small branches removed. However, when the object boundary representation is imprecise, e.g., a point cloud with noise and/or missing data, computing an accurate and reliable medial axis transformation is a more challenging task. In this work, we present a method towards a robust, accurate and compact representation for the medial axis transformation of a 2D point cloud (containing noise and missing data) using distance solution together with sparse optimization technique. Our algorithm consists of following four steps:

1. First, a signed distance function with respect to the noisy point cloud is computed by solving the Eikonal equation.
2. Second, an approximation of the signed distance function is obtained that can accurately recover the non-smooth ridge based on sparse optimization technique.
3. Third, the medial axis is extracted by locating the non-smooth ridge of the distance function with a new measure related to the derivative jump along the gradient direction of the distance function.
4. Finally, the medial axis is segmented into different branches, and a compact spline representation of each branch is obtained by fitting the discrete medial axis.

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