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Bivariate developmental relations between calculations and word problems: A latent change approach

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ABSTRACT

The relation between 2 forms of mathematical cognition, calculations and word problems, was examined. Across grades 2–3, performance of 328 children (mean starting age 7.63 [*SD* = 0.43]) was assessed 3 times. Comparison of *a priori* latent change score models indicated a dual change model, with consistently positive but slowing growth, described development in each domain better than a constant or proportional change model. The bivariate model including change models for both calculations and word problems indicated prior calculation performance and change were not predictors of subsequent word-problem change, and prior word-problem performance and change were not predictors of subsequent calculation change. Results were comparable for boys versus girls. The bivariate model, along with correlations among intercepts and slopes, suggest calculation and word-problem development are related, but through an external set of overlapping factors. Exploratory supplemental analyses corroborate findings and provide direction for future study.

1. Introduction

Mathematics comprises a variety of related branches focused on the study of quantities as expressed in numbers or symbols. In the primary grades, the major curricular focus is whole numbers, usually conceptualized as understanding number, performing calculations (CALC), and solving word problems (WP); in the intermediate-middle grades, the focus shifts to rational numbers and algebraic thinking, and the high school curriculum includes algebra, geometry, trigonometry, and calculus (Common Core State Standards Initiative, 2010; Geary et al., 2008; National Mathematics Advisory Panel, 2008). Yet, little is understood about how various aspects of mathematical cognition relate to: which aspects are shared or distinct and whether development in one domain influences development in others. Such understanding would provide insight into the nature of mathematics competence and guidance about how to optimize the design of curriculum and instruction.

The focus of the present study was connections between development of whole-number CALC and WP skill across grades 2–3. We focused on these grades because substantial progress occurs on multi-digit whole-number CALC and on WPs in this developmental period (Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014; Geary, 2011). Both forms of early mathematical cognition are essential. Competence with whole-number CALC is foundational for success with every subsequent form of mathematics. WP solving, the best school-age predictor of adult employment and wages (Every Child a Chance Trust, 2009), represents a major emphasis within almost every strand of the mathematics curriculum throughout school. Yet, connections between these two essential forms of mathematical cognition are not well understood.

With typical development of CALC skill, children enter school with rudimentary understanding of number, insights into basic addition-subtraction concepts, and skill in counting objects to solve single-digit problems (Geary, 1994). In kindergarten and first grade, most children achieve a reliable, full set of associations among phonological, visual, and semantic representations of numerals, and addition-subtraction concepts and skilled use of counting for solving arithmetic problems improve (e.g., Groen & Resnick, 1977). Across grades 1–3, use of efficient counting strategies produces repeated associations of simple arithmetic problems with answers to secure representations in long-term memory (Fuson & Kwon, 1992; Siegler & Robinson, 1982; Siegler & Shrager, 1984). This permits direct retrieval, freeing attentional resources and providing the foundation for more complex CALC (Fuchs et al., 2013; LeFevre & Morris, 1999). Such multi-digit CALC involve knowledge of procedural steps that differ in sequence and complexity by problem type and rely on deliberate decision

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making, constrained by understanding of the numeration system and the ability to monitor strings of procedural actions (Fuchs et al., 2014).

Less is known about how children develop facility with WPs. Moreover, prior work has not investigated how CALC and WP development is connected. What's known is that concurrent performance between domains is moderate to substantial (*r*s range from 0.35 to 0.71; e.g., Fuchs et al., 2008, 2010a, 2010b; Seethaler, Fuchs, Fuchs, & Compton, 2012). There are three explanations for this relation.

One possibility is that contextualizing CALC via stories supports understanding about CALC, and WP skill thus contributes to developing CALC skill. Some studies provide support for the hypothesis that contextualizing mathematics instruction with meaningful situations improves learning (Bottge, 1999; Cordova & Lepper, 1996; Maccini, Mulcahy, & Wilson, 2007), and this perspective is reflected in instruction designed to strengthen CALC in the primary grades (Bryant et al., 2011; Fuchs et al., 2013; Fuchs, Powell, et al., 2014). Yet, the tenability of assumptions underlying this hypothesis have been questioned (Anderson, Reder, & Simon, 1996), and we identified no studies testing whether contextualizing CALC problems with stories facilitates CALC performance or whether improved WP skill is a leading indicator of subsequent CALC performance.

A second possibility for the moderate to strong relation between CALC and WP skill is that CALC skill is foundational for WP solving. Solving WPs relies on CALCs. However, whereas a CALC problem is set up for solution, a WP requires students to process a narrative to build a problem model and then construct number sentence(s) for calculating the unknown quantity. Levine, Jordan, and Huttenlocher (1992) provided some evidence that CALC skill precedes WP solving. Testers presented a set of objects, then moved the objects into a box and transformed this hidden set by visibly adding or removing objects. Children then constructed an array showing the number in the hidden set. Despite success with this CALC task at age 4, most children could not solve analogous WPs problems until age 5. This suggests that facility with CALC precedes WP solving and that CALC are foundational to and are a leading indicator of WP solving. This is consistent with longitudinal correlational work, where initial CALC skill predicts WP outcomes (Fuchs et al., 2006; Swanson & Beebe-Frankenberger, 2004).

It is nevertheless also possible that Levine et al. (1992) at age 4 reflect child-level variables necessary to support successful WP solving, not adequately developed until age 5. One likely candidate, which emerges in the literature as active in WP solving, is language comprehension (Fuchs et al., 2008, 2010a, 2010b). Kintsch and Greeno (l985) described WP solving as demanding of language comprehension processes, and Cummins, Kintsch, Reusser, and Weimer (1988) provided corroborating data when they computationally modeled incorrect WP solving with incorrect math processes versus language comprehension processing errors. Results indicated that correct problem representation depended more on language comprehension; moreover, changing wording in minor ways dramatically affected accuracy. Some research identifies language comprehension as uniquely predictive of WP (not of CALC), while other external child-level factors, such as reasoning and working memory, are common across domains (Fuchs et al., 2010a, 2010b; Swanson, 2006). Common sources of variance in explaining individual differences across CALC and WP development speak to the third explanation for the moderate to strong relation between CALC and WP skill: CALC and WP development are related through an external set of overlapping factors.

The purpose of the present study was to deepen insight into the tenability of these explanations by directly investigating whether development in one domain influences the other. Based on Levine et al. (1992), along with studies indicating CALC skill predicts WP outcomes, our hypothesis was that CALC skill is foundational to WP solving (explanation #2). This hypothesis was, however, tentative because effects of WP solving on CALC development (explanation #1) have not been examined and because the literature reveals cognitive processes representing common external influences that may explain concurrent relations (explanation #3).

To consider these explanations, we assessed children's performance in both domains in fall and spring of second grade and spring of third grade. Relying on *a priori* latent change score models (McArdle, 2009), we first determined which of three models (constant change, proportional change, dual change) best describes development in each domain; we refer to these models as univariate models because only one outcome is considered at a time.

The constant change model, with a constant linear change term (slope), describes change as stable over time. In our specific case, this would mean that (on average) students grow at a steady rate over the course of grades 2 and 3 in terms of CALC and WP and that incremental change (during the next 6-month period as determined by our testing waves) is not dependent upon previous levels of performance.

By contrast, the proportional change model posits that change occurs incrementally, with change in one 6-month period proportional to performance at the preceding time. Substantively, this would mean that (on average) students' change in CALC and WP is related to their previous level of CALC and WP, respectively, with no constant or stable growth over the span of grades 2–3.

The dual change model describes development as a function of linear change and incremental change (see Table 1 for parameter/estimate interpretations). This would mean that change in CALC and WP has two influences, one that is constant or stable across the span of grades 2–3 and one that is dependent on (or

Table 1
Summary of parameter and estimate interpretations.

Parameter/estimate	Interpretation
σ_Y^2	Residual variance of time-specific Y scores
S_Y	Residual of time-specific Y scores
Y	Observed indicators of latent Y performance
L1–L4	Latent true scores
σ_I^2, intercept variance	Variability among students in predicted fall of grade 2 performance
I, intercept mean	Predicted level of performance at the fall of grade 2
β	Autoregressive/proportional change path predicting latent change as a function of the previous level of performance (above and beyond stable change)
Δ1–Δ3, latent change scores	Latent change between testing waves influenced by a constant/stable component and the previous latent score
$\sigma_{I,S}$, intercept and slope covariance	Relation between fall performance and rate of growth
σ_S^2, slope variance	Variability among students in rates of growth across grades 2 and 3
S, slope mean	Stable, constant change over the course of the study (above and beyond the proportional change); rate of growth
α	Loading for the latent slope factor, set to 1 for model identification