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Application of multilevel directional adaptive cross approximation technique for electromagnetic problems[☆]

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ABSTRACT

In this paper, a novel scheme is presented for forming the matrix equations of multilevel adaptive cross approximation (MLACA) algorithm. The main idea of the proposed technique is to use the directional grouping scheme to subdivide the far-field domain of MLACA algorithm. By using the grouping scheme, the far-field interaction domain can be divided into many cone structures. The matrix between the observation group and far-field group in the cone structure is low-rank, which meets the directional far-field requirement. At the same time, the near-field interaction matrices are formed by the SVD(T) method to further reduce the total memory requirements. With the given techniques, the memory requirement of the novel grouping scheme for the far-field is much less than half of traditional MLACA algorithm. Meanwhile, the memory requirement of the SVD(T) method for the near-field is only about one-third of direct filling.

1. Introduction

APPLICATION of the integral equation methods for solution of linear electromagnetic problems has many advantages. Only the surfaces of considered domains need to be discretized, open boundary problems pose no additional difficulties, and problems including motion can be treated elegantly. However, application of the integral equation methods leads to dense matrices. The memory requirements and computational costs are of $O(N^2)$, where N is the number of unknowns.

In recent years, there are many fast algorithms [1–2] have been proposed to reduce the pressure of memory requirements and computational costs of method of moments (MoM) [3]. The matrix compression technique is a kind of commonly used numerical method, which is a pure mathematical method. Therefore, it does not depend on the expansion of Green's function. It can easily be applied to analyze the complex targets electromagnetic problems. In [4–5], a "butterfly" decomposition method called multilevel matrix decomposition algorithm (MLMDA) is proposed to analyze the scattering from electrically large objects, which utilizes the idea of equivalent sources to gain an efficient matrix compression technique. Multilevel UV method is proposed in [6–7], which makes the rank table for the problems and then applies the rank table to decompress the impedance matrix. References [8] introduce an efficient matrix compression technique named H-matrix, whose numerical complexity can be reach to $O(N\log N)$. In [15], an efficient form of ACA algorithm [14,11,12] is introduced, which utilizes the QR factorization and singular value decomposition to further compress the sub-matrices of ACA algorithm. The multilevel adaptive cross approximation (MLACA) is also an efficient matrix compression technique for analyzing the boundary value problems. The matrices of MLACA algorithm is decomposed by extracting the rank of rows and columns. Two improved forms of the MLACA [9,10] are introduced in [13] and [16], respectively, which are much more efficient than traditional MLACA algorithm [9,10] for large targets. A fast direct method based on ACA algorithm is proposed in [17] for analyzing the electrically large integral equations for problem sizes to 1 M unknowns.

Although the matrix compression method has so many advantages, it consumes a lot of time in the matrix filling process. In this paper, the matrix filling process of MLACA algorithm is analyzed and a novel tree structure called directional grouping scheme [18–21] is applied to reduce the computational time of the matrix filling process. It utilizes the directional grouping scheme to divide the far-field interaction area into a lot of cones. The interaction matrix formed by the observation box and far-field cone is low rank, which meets the directional far-field requirement [19]. The interaction matrices can be filled by matrix compression

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algorithm efficiently. By using this technique, an efficient matrix filling process is obtained. The numerical results show that the proposed technique can reduce the computational time and memory of matrix filling process of MLACA algorithm significantly, with excellent accuracy. Meanwhile, the near-field interaction matrices can also be represented by low rank compression technique, and its rank is several times larger than that of the far-field matrix, but it is much smaller than the dimensions of the near-field interaction matrix. In this paper, the near-field interaction matrices are compressed by SVD(T) method [22] to further reduce the total memory requirements.

2. Multilevel directional adaptive cross approximation

To obtain the matrix representation of traditional MLACA algorithm, an efficient approach is proposed to partition the problem domain into smaller boxes. The partition scheme is same as that of multilevel fast multipole algorithm (MLFMA) [1]. To construct a cluster tree, we start from the root cluster which is the full index set. We then find a disjoint partition of the index set and use this partition to create children clusters. We continue the partition process until the lowest-level square boxes, whose electrical size is less than 1/10 wavelength. The box in the high level is termed “parent” and the box that is contained in the “parent” box is termed “children.” The possible far-field action boxes for an observation box at the “children” level is limited in the near-field of its “parent” level, which is shown in Fig. 1.

According to the Fig. 1, there may be 27 far-field action groups for the “A” box in the two dimensional case. At the same time, there are 189 possible far-field boxes in the three dimensional case. The interaction matrices between the observation box and the far-field action boxes are filled by ACA algorithm. The number of the interaction matrices is very large, so that the matrix filling time of traditional MLACA algorithm is relatively long.

2.1. *The single level of directional adaptive cross approximation*

In order to improve the matrix filling process, a novel tree structure called directional grouping scheme [18–19] is used in this paper. It divides the far-field interaction area into many cones. The problem domain is first encapsulated in a smallest square box, and then a hierarchical subdivision of the square box is constructed until the box’s size of the lowest-level is less than 1/10 wavelength, similar to the traditional tree structure of MLFMA. Then, the far-field interaction area of each level is partitioned into a group of cone regions. In Fig. 2(a), the form of directional grouping scheme for an observation box in the two dimensional case is given. Meanwhile, the three dimensional case is given in Fig. 2(b).

Here, w denotes the box’s size at the level. l denotes the number of the cone regions, and the $1/w$ denotes the angle of cone. The cone regions at each level are in the same size ($O(1/w)$), which are used to divided the far-field area of the observation box. The far-field action parts of the observation box are in the distance of w^2 away. In Fig. 2, the “Y” denotes the observation box, and the “X” denotes the far-field action boxes of the observation box. The interaction matrix formed by the observation box and far-field cone is low rank, which meets the directional far-field requirement [19]. The interaction matrices can be filled by matrix compression algorithm efficiently. The ACA is utilized to fill interaction matrices, which constructs the interaction matrix through the following product form,

$$[Z]^{m\times n} = [U]^{m\times r}[V]^{r\times n} \qquad (1)$$

where $[Z]^{m\times n}$ denotes the interaction matrix between the observation box and far-field cone, r denotes the rank of the interaction matrix ($r \ll \min(m, n)$). It needs about $O(Nr)$ (where N denotes the number of unknowns) to store the sub-matrices formed by the single level of directional adaptive cross approximation (DACA) algorithm, which is much less than that of the MoM.

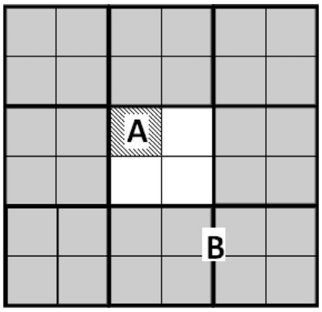
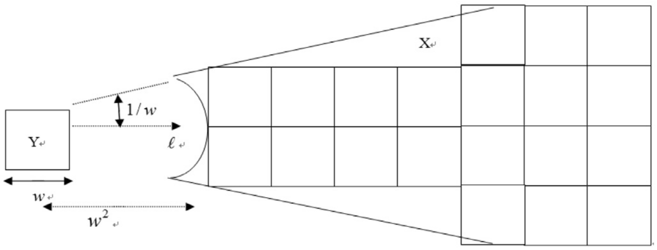
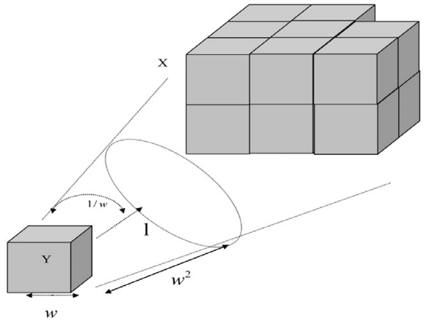


Fig. 1. The far-field action boxes for an observation box in the two dimensional case.



(a) The form of directional grouping scheme in the two dimensional case.



(b) The form of directional grouping scheme in the three dimensional case.

Fig. 2. (a) The form of directional grouping scheme in the two dimensional case. (b) The form of directional grouping scheme in the three dimensional case.

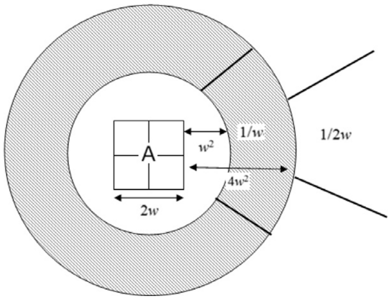


Fig. 3. The form of multilevel directional grouping scheme.

2.2. *The multilevel of directional adaptive cross approximation*

Fig. 3 shows the multilevel directional grouping scheme in two dimensional case. It can be seen from the figure, the grey domain is the possible far-field interaction area of the box A’s children, which is the near-field of the box A. The far-field interaction area of the box A is in the distance of $4w^2$ away. And so on, the far-field interaction area of the