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Effects of constructing, critiquing, and revising arguments within university classrooms

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ABSTRACT

Fifty-seven students in mathematics content and secondary mathematics methods classrooms at four universities participated in an instructional sequence to communally generate criteria defining a valid proof. Participants were asked to complete a proof-related task before class, work together in small groups to evaluate fellow students' arguments, communally agree upon criteria for evaluating said arguments based on their evaluations, and then rate and revise their original argument to satisfy the communal criteria after class. We used a modified analytical categorization of arguments to compare students' before-class and after-class work. Results demonstrated that students' self-rating aligned positively with the argument categories, but two critical challenges were also observed: (1) students' self-rating of their arguments was not positively correlated with the class-acceptance rate of those arguments indicating that individual and communal perceptions of proof varied; and (2) students struggled to revise their arguments to align with the communal criteria.

1. Introduction

In recent years, there has been an increased attention to proofs in mathematics classrooms because proving is fundamental to doing mathematics and communicating mathematical ideas (Stylianides, 2007). Research investigating secondary school and university students' conceptions of proof has illuminated complexities with the teaching and learning of proof (Harel & Sowder, 2007; Knuth, Choppin, & Bieda, 2009; Morris, 2007; Weber, 2010). Secondary school and university students typically observe their teacher presenting a polished and completed proof and are asked to replicate and assimilate strategies on similar problems (Stylianou, Blanton, & Knuth, 2009). When the instructor is the primary presenter and evaluator of mathematical arguments, students are limited in their opportunities to make sense of how to construct or critique arguments on their own (Harel & Rabin, 2010). This method of proof instruction has caused students to believe that their teacher is the sole authority when it comes to judging the validity of a proof (Harel & Sowder, 1998), and is considered a contributing factor as to why university students' exhibit difficulties with constructing and evaluating arguments (Bleiler, Thompson, & Krajčevski, 2014; Morris, 2007; Ko & Knuth, 2013; Selden & Selden, 2003; Weber, 2001, 2010).

To address this instructional issue, students require opportunities to actively participate in the construction and evaluation of

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arguments throughout the learning process (Bleiler, Ko, Yee, & Boyle, 2015; Selden & Selden, 2015; Stylianides & Stylianides, 2009a, 2009b). In fact, proof "involves the subjective negotiation of not only the concepts concerned, but implicitly also of the criteria for an acceptable argument" (de Villiers, p. 22, 1990). Producing and evaluating arguments is especially important because the Common Core State Standards for Mathematical Practice suggest that K-12 students should be able to construct viable arguments and critique the reasoning of others (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Preservice secondary mathematics teachers require these opportunities as well because they are setting the foundation for future students' conceptions of what counts as proof. Therefore, university classrooms need to provide students with opportunities to actively participate in proof construction and evaluation rather than the teacher acting as the only authority (Harel & Rabin, 2010; Ko, Yee, Bleiler-Baxter, & Boyle, 2016; Stylianides & Stylianides, 2009a).

Decentralizing authority away from the teacher can be difficult because students will require criteria to define a proof. For example, if a student is asked to construct a proof and they produce an empirical argument, then it is not clear why they believe their empirical argument is proof (Stylianides & Stylianides, 2009a). However, if the student is able to evaluate and justify an argument as empirical, then there is opportunity to promote meaningful conversation about the validity of the argument. Generating such meaningful conversation requires students to engage in communal evaluation of arguments. Most research on proof evaluation is not aimed at capturing students' understanding of how to evaluate arguments. Instead students are expected to evaluate arguments based on their prior knowledge of proof (e.g., Bleiler et al., 2014; Ko & Knuth, 2013; Selden & Selden, 2003). To date, we have found only Stylianides and Stylianides (2009a) explicitly shared criteria that was developed between an instructor and students in the classroom community. However, they did not have the students use the criteria to revise their constructed arguments. Therefore, there is a need to provide insight about how university students use criteria they developed to evaluate and revise their own argument.

To support university students in developing such a communal list of criteria for what constitutes proof, we designed an instructional sequence that included a before-class, during-class, and after-class activity. In this article, we share the classroom communal proof criteria, a selected subset of participants' arguments, how the student author of the argument evaluated his/her own work, how the class evaluated the individual's arguments, how the researchers coded the selected arguments, and how the students revised their arguments. More specifically, this study is guided by the following research questions:

(1) How did the class's acceptance of a single student's argument compare with the student's self-rating of their argument?
(2) How did students' self-rating of their arguments across the class criteria compare with the arguments' mode of argumentation (e.g., empirical argument, generic example, deductive demonstration)?
(3) How did students' arguments for the Sticky Gum Problem (Fig. 1) differ before and after developing a class-based criterion for what counts as proof?

We hypothesized that there would be a significant positive correlation between the class acceptance rate of a given student's argument and that student's individual self-rating of his/her argument after developing a communally negotiated criteria. Furthermore, we anticipated that a student's revised argument would align with the class criteria.

2. Perspective of proof

One perspective of proof is that "a mathematical proof is a formal and logical line of reasoning that begins with a set of axioms and moves through logical steps to a conclusion" (Griffiths, 2000, p. 2). This definition adequately describes a formal individual process of writing a proof, but it ignores the negotiation aspect of validating proofs that occur within a mathematics community. We draw upon Stylianides' (2007) definition of proof because it also highlights the communal aspects of proof:

Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

1. It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further

The Sticky Gum Problem

Ms. Hernandez came across a gumball machine one day when she was out with her twins. Of course, the twins each wanted a gumball. What's more, they insisted on being given gumballs of the same color. The gumballs were a penny each, and there would be no way to tell which color would come out next. Ms. Hernandez decides that she will keep putting in pennies until she gets two gumballs that are the same color. She can see that there are only red and white gumballs in the machine.

1) Why is three cents the most she will have to spend to satisfy her twins?
2) The next day, Ms. Hernandez passes a gumball machine with red, white, and blue gumballs. How could Ms. Hernandez satisfy her twins with their need for the same color this time? That is, what is the most Ms. Hernandez might have to spend that day?
3) Here comes Mr. Hodges with his triplets past the gumball machine in question 2. Of course, all three of his children want to have the same color gumball. What is the most he might have to spend?
4) Generalize this problem as much as you can. Vary the number of colors. What about different size families? Prove your generalization to show that it always works for any number of children and any number of gumball colors.

Fig. 1. The Sticky Gum Problem (Fendel, Resek, Alper, & Fraser, 1996).