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Assessment of third invariant elasto-plastic models: Mathematical aspects, numerical strategies and comparative results

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ABSTRACT

This contribution presents a study of Hosford's and Gao et al.'s elasto-plastic models each influenced by the third invariant of the deviatoric stress tensor (J_3) in their formulations. The first stage of this work is a review of the concepts of Mechanics of Materials and Constitutive Modeling Theory. Subsequently, Hosford's and Gao et al.'s models with isotropic hardening are assessed taking into account their mathematical and numerical aspects. Their yield surface's shape and consequent convexity issues are investigated through a preliminary analysis of the effects that the third invariant (J_3) has on their formulations. A return mapping algorithm is then proposed and tested for Hosford's and Gao et al.'s elasto-plastic models based on the operator splitting methodology. The proposed algorithm was implemented through an implicit numerical integration method in an academic finite element environment, along with its consistent tangent matrix. Finally, the numerical results obtained from the proposed models are compared to experimental data available in the literature. This research assesses the performance and precision of the proposed constitutive formulations to correctly describe the mechanical behavior of an aircraft aluminum alloy and 1045 alloy steel under realistic stress and strain fields. In order to do so, different specimens capable of generating distinct stress states within high and low stress triaxiality regions are taken into account. As a result, this work demonstrates that Gao et al.'s yield criterion may generate non-convex yield surfaces when the influence of the third invariant is especially strong. In addition, it is observed that Hosford's constitutive model presents better agreements between the obtained numerical results and the experimental data collected from the literature. The most important finding however is that the third invariant should be essential to any formulation.

1. Introduction

In order to precisely and accurately describe the elasto-plastic behavior of ductile metallic materials, constitutive formulations have been extensively studied over the past two centuries. The need for a correct prediction of the moment and location of the onset of ductile fractures is a major focus for high performance industrial sectors and has been researched both in academic and industrial environments. Despite years of research, many questions remain unresolved, challenging researchers.

Aiming to develop more competitive products, with enhanced features for better performance, durability, ease of manufacturing and lower cost, the aerospace, automotive, naval and arms industries have been employing increasingly advanced scientific methodologies. Their goal is to achieve efficient and cost effective production whilst the proper functionality of the product is ensured. As an example, Walp et al. [51] point out the significant commitment of the automotive industry to the quest for reducing weight and the amount of material used in vehicle fabrication, especially in structures such as chassis and bodies, without reducing important characteristics such as rigidity, performance and market competitiveness. In order to achieve such improvements, elasto-plastic models capable of precisely describe the mechanical behavior of ductile materials are used to identify the correct location and moment that a crack initiate in the material, which may eventually evolve and propagate until the complete fracture of the material.

One of the most commonly used formulations for describing the elasto-plastic behavior of ductile materials is based on the equivalent stress, i.e., based on the second invariant of the deviatoric stress tensor, J_2. This contitutive formulation, widely known as von Mises' model (1913), proposes that plastic yielding begins when the second invariant of the stress deviator tensor, J_2, reaches a critical value. However, it is experimentally observed that the application of this classic formulation cannot be generalized due to its limited description of the mechanical behavior of ductile materials, as this model does not present the same precision for a wide range of materials, especially modern alloys ([2];

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Driemeier, 2015).

In the study of elasto-plasticity some parameters are commonly referred to in the definition of a material point's stress state. Along with the second invariant of the deviatoric stress tensor (J_2), the so-called hydrostatic stress (p) and the third invariant of the deviatoric stress tensor (J_3) are among the most important parameters and concepts emphasized in recent researches in plasticity. Hydrostatic stress (p) has been extensively studied over the past decades due to its influence on ductility of metallic materials. Bridgman [13] conducted experiments with different types of steel and concluded that the deformation to fracture is influenced by the hydrostatic stress effect, i.e., higher hydrostatic stress levels result in greater elongations until material failure occurs. According to Rice & Tracey [45], this parameter acts as a regulatory factor in void growth rates. It contributes to void closing under compression, and to void nucleation and growth under tension conditions. Moreover, according to Hancock & Mackenzie [26], the onset of ductile fracture occurs at locations where the levels of hydrostatic stress are maximized.

Hydrostatic stress is commonly introduced in mathematical formulations through its normalization with respect to an equivalent stress, which characterizes the general stress state through a scalar. Such representation is termed stress triaxiality (η). As an effect, when added to constitutive formulations, the triaxiality ratio controls the size of the elastic domain/yield surface of a ductile material. However, as presented by Barsoum and Faleskog [7,8], this parameter is not sufficient to characterize material's ductility, especially in situations of predominant shear loading, i.e. low stress triaxiality conditions.

In this sense, in conditions where shear is predominant, the third invariant of the stress deviator tensor (J_3) presents a more significant correction for the prediction of ductile damage and failure. For instance, the addition of this parameter to constitutive formulations improves their ability to describe certain characteristics of the process of ductile failure, such as the evolution of the shape of voids in the material, the formation of shear bands, the change in ductility under tension and torsion/shear regimes, and the transition between failure modes [4,7–9,17]. Often these effects are introduced in the mathematical formulation of yield criteria through the so-called Lode angle (θ), which is a function of the normalized third invariant of the stress deviator tensor (ξ). Thereby, when added to mathematical formulations, the third invariant changes the shape of the material's yield surface [2,6].

The impact of these two parameters, hydrostatic stress and third invariant of the deviatoric stress tensor, on the description of the mechanical behavior of ductile materials has received great attention. Detailed studies about the influence of these parameters in the constitutive formulation of elasto-plastic models and Damage Mechanics were proposed by several authors [2,19,41,24,31,17,37,39,20]. According to Bai [2], for loading conditions with shear components, the Load angle parameter represents a stronger effect in the description of the mechanical behavior of elasto-plastic materials than the stress triaxiality. Therefore, in such load conditions, the improvement in precision obtained by the introduction of the third invariant effect in constitutive modelling (commonly added to constitutive models through the Lode angle) is more significant than that obtained by the introduction of the triaxiality ratio.

Numerous other experimental analyzes were conducted by examining the effects of hydrostatic stress and stress triaxiality on ductile fracture. Richmond and Spitzig [46] were the first to study the effects of hydrostatic stress in aluminum alloys' plastic flow. More recently, Bao & Wierzbicki [4,5] conducted an extensive experimental program with Al2024-T351 aluminum alloy, using eleven different specimens. According to them, ductile failure occurs due to different failure mechanisms depending on stress triaxiality levels. Furthermore, according to Kim et al. [32] and Gao & Kim [23], different stress states with the same triaxiality stress present different void growth and coalescence behavior.

In this regard, in order to make a distinction between stress states that have the same stress triaxiality ratio level the Lode angle parameter should be introduced. Hence, the stress field is uniquely determined [23]. In the following decade, Bardet [6] proposed a methodology to describe the dependence on the Lode angle in certain constitutive models. Wilson [52] utilized 2024-T351 aluminum alloy notched specimens under tensile loading conditions to verify the importance of this effect.

Mirone & Corallo [41] gathered experimental data of three different types of alloy steel and pure copper from literature and conducted numerical simulations, considering smooth and notched specimens, to analyze the effects of the stress triaxiality and Lode angle on the description of mechanical behavior and failure of ductile materials. The authors concluded that the Lode angle parameter presents an unimportant effect on the damage process, but a significant impact on hardening evolution. In contrast, stress triaxiality poses a great influence on the damage process and negligible effect on hardening. Driemeier et al. [19] carried out an experimental program to analyze the combined effects of the equivalent stress, stress triaxiality and Lode angle on the elasto-plastic behavior of an aircraft aluminum alloy. The hypothesis that the Lode angle and hydrostatic stress parameters influence yield surface is still very difficult to verify experimentally [31]. Nevertheless, Brünig et al. [17] relate a decrease in material ductility to an increase of the triaxiality ratio and also indicate that in addition to the equivalent stress, the triaxiality ratio and Lode angle represent the most important factors controlling the onset and evolution of damage and ductile fracture.

Ductile fracture is a local phenomenon and the state of stress and strain in the expected fracture locus must be determined with accuracy. Classical formulations, such as von Mises [50] and Tresca [49], do not present the required precision to satisfactorily describe the elasto-plastic behavior of modern alloys under wide ranges of stress triaxiality. Therefore, such applications demand more refined and precise constitutive models [3].

Following the trend of the past decades, two constitutive models dependent on the third invariant of the deviatoric stress tensor are covered in this contribution, considering the addition of this parameter to the plastic flow rule of ductile materials. The yield criteria proposed by Hosford [29] and Gao et al. [24] are taken into account to mathematically formulate the constitutive models proposed in this work.

2. Theoretical aspects

2.1. Ductile fracture

When ductile materials are loaded within its plastic regime the relationship between stress and strain fields is nonlinear. Unlike the material's elastic behavior, where Hooke's law establishes a linear and explicit relationship between stress and strain, in the plastic regime there is no such direct relation, making the task of mapping the stress and strain fields harder and only possible through the adoption of constitutive models. Aiming to determine the correct location and moment for the fracture onset, many researches have proposed theories to describe the plastic behavior of materials, some of the most famous are Tresca [49], von Mises [50] and Drucker-Prager [21].

Empirical evidences show that ductile failure of metallic materials occurs due to the evolution of micro-defects such as micro-voids and shear bands [26,40,45]. The ductile fracture phenomenon is usually induced by primary inclusions and second phase particles [14], where micro-defects nucleation takes place through decohesion of inclusions (or second-phase particles) from the surrounding metal matrix [1,43], or through the fracture of inclusions [25].

Gao & Kim (2003 e [23]) present the steps for the evolution process of micro-defects: (1) first, nucleation of micro-defects occurs due to fracture or dislocation of second-phase inclusions, then (2)