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An efficient high order plane wave time domain algorithm for transient electromagnetic scattering analysis

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ABSTRACT

An efficient high order plane wave time domain algorithm is presented for analyzing the transient scattering from three dimensional electrically large conducting objects. This method uses a set of hierarchical divergence-conforming vector basis functions to accurately represent the current distribution on the perfect electrically conducting (PEC) surface. The higher order functions can significantly reduce the number of unknowns without compromise on the accuracy. The time domain combined field integral equation (TD-CFIE) is then discretized using the hierarchical divergence-conforming vector basis functions and shifted Lagrange polynomial functions in spatial and time domain, respectively. The final matrix equation can be accelerated using the plane wave time domain (PWTD) algorithm. Finally, a parallel algorithm that can execute on a distributed-memory parallel cluster is developed, which provides an appealing avenue for analyzing the transient scattering from three-dimensional electrically large complex PEC objects. Numerical examples are given to demonstrate the accuracy and efficiency of the method.

1. Introduction

With the continuous improvement of late time instability occurring in the marching-on-in-time (MOT) based time domain integral equation (TDIE) method [1–7], the TDIE method has been of considerable interest in the computational electromagnetic (CEM) community for their appealing avenue in analyzing transient and broadband electromagnetics problems [8–10]. The computational complexity and memory requirement for the analysis of transient scattering from an object using the classical MOT algorithm are $O(N_t N_s^2)$ and $O(N_s^2)$, respectively, where N_t and N_s are the numbers of temporal and spatial basis functions. The high computational burden greatly hinders the performance of TDIE algorithm to analyze the large-scale electromagnetic scattering problems. A great deal of attention has been focused on the development of time-domain fast solvers, among which, the plane wave time domain (PWTD) algorithm is the most representative one. Relying on a Whittaker-type expansion of transient fields in terms of propagating plane waves, the computational cost and memory requirements of the PWTD algorithm scale as $O(N_t N_s \log^2 N_s)$ and $O(N_s^{1.5})$, respectively [11–13]. However, the PWTD algorithm can only reduce the computational complexity of the classical MOT algorithm. There is another approach that directly reduces the number of unknowns to be solved by using the higher order spatial basis functions [14–18]. If we combine the fast algorithm with the high order basis function in the framework of the MOT based TDIE, this will greatly reduce the computational time and memory consumption of the TDIE method, from the perspective of reducing the number of unknowns and the computational complexity. The similar approach has been extensively studied in the frequency domain method of moments (MOM) [21–24]. However, few works has been proposed in the time domain.

In this paper, an efficient high order PWTD algorithm is presented for analyzing the transient scattering from three dimensional electrically large conducting objects. We first use a set of hierarchical divergence-conforming vector basis functions [19–20] and shifted Lagrange polynomial functions to discrete the time domain combined field integral equation (TD-CFIE). The hierarchical divergence-conforming vector basis functions can accurately represent the current distribution on the PEC surface and significantly reduce the number of unknowns without compromise on the accuracy. Then the final matrix equation can be accelerated using the PWTD algorithm. Finally, a parallel high order PWTD algorithm is developed, which provides an appealing avenue for analyzing the transient scattering from three-dimensional electrically large complex PEC objects. The described solver executes on a distributed-memory parallel cluster and uses the message passing interface (MPI) paradigm to communicate data between processors. Numerical examples are given to demonstrate the accuracy and efficiency of the proposed method.

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2. Formulation

This section describes the implementation of proposed method in detail. Section 2.1 derives and establishes the time-domain electric field, magnetic field and combined field integral equation for solving the transient electromagnetic characteristics from PEC object. Section 2.2 describes the marching on-in-time scheme for solving these equations and the specific spatial basis functions adopted in this paper. Then, the high order PWTD algorithm is introduced in Section 2.3. Finally, a parallel implementation of the scheme is described in Section 2.4.

2.1. Time domain integral equation

When an arbitrary shaped PEC object residing in free space with permeability μ_0 and permittivity ε_0 is illuminated by a transient incident field $\mathbf{E}^{inc}(\boldsymbol{r},t)$ and $\mathbf{H}^{inc}(\boldsymbol{r},t)$. The induced current $\mathbf{J}(\mathbf{r}, t)$ on surface S generates a scattered field $\mathbf{E}^{sca}(\boldsymbol{r},t)$ and $\mathbf{H}^{sca}(\boldsymbol{r},t)$ which can be characterized as

$$\mathbf{E}^{sca}(\boldsymbol{r},t) = -\frac{\mu_0}{4\pi}\iint_s dS\,\frac{1}{R}\frac{\partial \mathbf{J}(\mathbf{r}',\tau)}{\partial t} + \frac{\nabla}{4\pi\varepsilon_0}\iint_s dS\int_0^{\tau}\frac{\nabla'\cdot\mathbf{J}(\mathbf{r}',t')}{R}dt' \tag{1}$$

$$\mathbf{H}^{sca}(\boldsymbol{r},t) = \nabla\times\frac{1}{4\pi}\iint_s dS\,\frac{\mathbf{J}(\mathbf{r}',\tau)}{R} \tag{2}$$

where $\mathbf{J}(\mathbf{r}, t)$ denotes the induced current on the PEC surface S, $\boldsymbol{r}$ and $\boldsymbol{r}'$ represent the observation point and source point, respectively, $R=|\boldsymbol{r}-\boldsymbol{r}'|$, c is the speed of light in free space, $\tau=t-R/c$ is the retarded time.

According to the boundary condition on the PEC surface:

$$\hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\left[\mathbf{E}^{inc}(\mathbf{r},t)+\mathbf{E}^{sca}(\mathbf{r},t)\right]\Big|_S = 0 \tag{3}$$

$$\hat{\mathbf{n}}(\mathbf{r})\times\left[\mathbf{H}^{inc}(\mathbf{r},t)+\mathbf{H}^{sca}(\mathbf{r},t)\right]\Big|_S = \mathbf{J}(\mathbf{r},t) \tag{4}$$

we can get the time domain electric and magnetic field integral equations:

$$\begin{aligned}&\hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{E}^{inc}(\boldsymbol{r},t)\\ &= \hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\frac{\mu_0}{4\pi}\iint_s dS\,\frac{1}{R}\frac{\partial\mathbf{J}(\mathbf{r}',\tau)}{\partial t}\\ &\quad-\hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\frac{\nabla}{4\pi\varepsilon_0}\iint_s dS\int_0^{\tau}\frac{\nabla'\cdot\mathbf{J}(\mathbf{r}',t')}{R}dt'\\ &= \boldsymbol{L}_e\{\mathbf{J}(\mathbf{r},t)\}\end{aligned} \tag{5}$$

$$\begin{aligned}&\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{H}^{inc}(\boldsymbol{r},t)\\ &= \frac{\mathbf{J}(\mathbf{r},t)}{2}-\hat{\mathbf{n}}(\mathbf{r})\times\frac{1}{4\pi}\iint_s dS\,\nabla\times\frac{\mathbf{J}(\mathbf{r}',\tau)}{R} = \boldsymbol{L}_h\{\mathbf{J}(\mathbf{r},t)\}\end{aligned} \tag{6}$$

Then the time domain combined field integral equation (TD-CFIE) can be derived as a linear combination of the time domain electric and magnetic field integral equations:

$$\begin{aligned}&\mathbf{V}_c\left\{\mathbf{E}^{inc}(\mathbf{r},t),\mathbf{H}^{inc}(\mathbf{r},t)\right\}\\ &= \alpha\hat{\mathbf{n}}(\mathbf{r})\times\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{E}^{inc}(\mathbf{r},t)+(1-\alpha)\eta\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{H}^{inc}(\mathbf{r},t)\\ &= \alpha\boldsymbol{L}_e\{\mathbf{J}(\mathbf{r},t)\}+\eta(1-\alpha)\boldsymbol{L}_h\{\mathbf{J}(\mathbf{r},t)\}\\ &= \boldsymbol{L}_c\{\mathbf{J}(\mathbf{r},t)\}\end{aligned} \tag{7}$$

where $\hat{\mathbf{n}}(\mathbf{r})$ is the outward directed unit normal, η is the wave impedance of the free space. $\alpha$is a positive real constant. The EFIE and the MFIE can be derived from the CFIE by setting $\alpha=1$ and $\alpha=0$, respectively. So the discussions hereafter will focus on the CFIE.

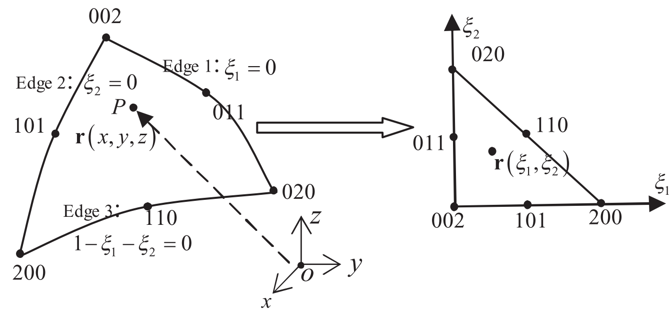

Fig. 1. The mapping relationship between the quadratic curvilinear triangle and the parameter triangle.

2.2. Discretization and marching on-in-time scheme

To solve the above equations numerically, the PEC surface S is discretized into curved triangular elements firstly. Then, the surface current density $\mathbf{J}(\mathbf{r}, t)$ can be expanded in terms of a set of vector spatial basis functions and scalar temporal basis functions as

$$\mathbf{J}(\mathbf{r},t)\cong\sum_{n=1}^{N_s}\sum_{l=1}^{N_t}I_n^l\boldsymbol{f}_n(\mathbf{r})T_l(t) \tag{8}$$

Here I_n^l are the unknown coefficients. N_t and N_s denote the number of time steps and spatial basis functions, respectively.

Substituting (8) into (7), Galerkin testing the Eq. (7) with the surface basis function $\boldsymbol{f}_m(\mathbf{r})$ and point matching at time $i\Delta t$, leads to the system of equations that can be represented explicitly in a matrix form as

$$\mathbf{Z}^0\mathbf{I}^i = \mathbf{V}^i-\sum_{j=1}^{i-1}\mathbf{Z}^{i-j}\mathbf{I}^j \tag{9}$$

The expression of matrix elements is

$$\left[Z^{i-j}\right]_{mn} = \left\langle\boldsymbol{f}_m(\mathbf{r}),\boldsymbol{L}_c\left\{\boldsymbol{f}_n(\mathbf{r})T_{i-j}(t)\right\}\right\rangle \tag{10}$$

$$\left[V^i\right]_m = \left\langle\boldsymbol{f}_m(\mathbf{r}),\mathbf{V}_c\left\{\mathbf{E}^{inc}(\mathbf{r},t),\mathbf{H}^{inc}(\mathbf{r},t)\right\}\right\rangle\Big|_{t=t_i} \tag{11}$$

Here, the spatial basis functions are the higher order hierarchical basis functions defined on curved parametric triangles. We use software ANSYS to generate the quadratic curvilinear elements with six nodes [27]. Fig. 1 shows the mapping relationship between the quadratic curvilinear triangle and the parameter triangle. The quadratic curvilinear triangle can be transformed into the planar parameter triangle using the transformation

$$\begin{aligned}\mathbf{r}(x,y,z) &= \mathbf{r}_{200}\xi_1(2\xi_1-1)+\mathbf{r}_{020}\xi_2(2\xi_2-1)\\ &\quad+\mathbf{r}_{002}\xi_3(2\xi_3-1)+\mathbf{r}_{110}4\xi_1\xi_2+\mathbf{r}_{011}4\xi_2\xi_3+\mathbf{r}_{101}4\xi_1\xi_3\end{aligned} \tag{12}$$

Based on the parametric triangular patch, higher order hierarchical vector basis functions can then be constructed. The lowest-order vector basis functions are the curvilinear RWG functions and the specific expression is

$$\begin{aligned}\boldsymbol{f}_{1,0}^e(\mathbf{r}) &= \frac{1}{J}\left[(\xi_1-1)\frac{\partial\mathbf{r}}{\partial\xi_1}+\xi_2\frac{\partial\mathbf{r}}{\partial\xi_2}\right]\\ \boldsymbol{f}_{2,0}^e(\mathbf{r}) &= \frac{1}{J}\left[\xi_1\frac{\partial\mathbf{r}}{\partial\xi_1}+(\xi_2-1)\frac{\partial\mathbf{r}}{\partial\xi_2}\right]\\ \boldsymbol{f}_{3,0}^e(\mathbf{r}) &= \frac{1}{J}\left[\xi_1\frac{\partial\mathbf{r}}{\partial\xi_1}+\xi_2\frac{\partial\mathbf{r}}{\partial\xi_2}\right]\end{aligned} \tag{13}$$

where J is the Jacobian. The first subscript denotes the edge and the second subscript denotes the order of the polynomials. The superscript denotes that they are edge-based basis functions.

The higher order bases functions are obtained by forming the product of the curvilinear RWG basis functions with a set of polynomial functions. Here we give the edge-based and face-based basis functions up to