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Formulation of solid-shell finite elements with large displacements considering different transverse shear strains approximations

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ABSTRACT

This work presents a general formulation and implementation in solid-shell elements of the refined zigzag theory and the trigonometric shear deformation theory in an unified way. The model thus conceived is aimed for use in the analysis, design and verification of structures made of composite materials, in which shear strains have a significant prevalence. The refined zigzag theory can deal with composite laminates economically, adding only two nodal degrees of freedom, with very good accuracy. It assumes that the in-plane displacements have a piece-wise linear shape across the thickness depending on the shear stiffness of each composite layer. The trigonometric theory assumes a cosine variation of the transverse shear strain. A modification of this theory is presented in this paper allowing its implementation with C^0 approximation functions. Two existing elements are considered, an eight-node tri-linear hexahedron and a six-node triangular prism. Both elements use a modified right Cauchy-Green deformation tensor $\overline{\mathbf{C}}$ where five of its six components are linearly interpolated from values computed at the top and bottom surfaces of the element. The sixth component is computed at the element center and it is enhanced with an additional degree of freedom. This basic kinematic is improved with a hierarchical field of in-plane displacements expressed in convective coordinates. The objective of this approach is to have a simple and efficient finite element formulation to analyze composite laminates under large displacements and rotations but small elastic strains. The assumed natural strain technique is used to prevent transverse shear locking. An analytic through-the-thickness integration and one point integration on the shell plane is used requiring hourglass stabilization for the hexahedral element. Several examples are considered on the one hand to compare with analytical static solutions of plates, and on the other hand to observe natural frequencies, buckling loads and the non-linear large displacement behavior in double curved shells. The results obtained are in a very good agreement with the targets used.

1. Introduction

The development and use of solid-shell elements have notably increased in the last decade. Particularly by the use of enhanced assumed strain (EAS) techniques in elements with reduced integration on the shell plane (8-node elements). The solid-shell elements have important advantages compared with shell elements as they allow to use three-dimensional constitutive relations, to get rid of rotational degrees of freedom, to modelize geometrical details and boundary conditions more faithfully, to deal with contact conditions on the real contact external surfaces, etc. Unfortunately this better geometric representation involves a greater computational cost for the through-the-thickness numerical integration. Solid-shell elements behave similar to shells elements based on the first order shear deformation theory (FSDT) as they naturally include the transverse shear strains, although the plane stress condition is imposed in an integral sense and not point-wise as shell elements do.

According to the properties of the composite laminates and the expected accuracy (in terms of stresses or displacements), different approaches are considered for the structural analysis. For through-the-thickness highly heterogeneous laminates, the classical laminated plate theory (CLPT) leads to poor predictions. Similar unacceptable results are obtained with the FSDT even if suitable shear correction factors (SCF) are used to include the effect of the through-the-thickness heterogeneity. The main drawbacks are derived from the assumption of linear displacement across the thickness, which cannot intrinsically

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satisfy the interlaminar shear stress continuity (IC) and the surface conditions prescribed by the equilibrium equations [6]. The CLPT and FSDT are only advisable when the length-to-thickness ratio is high and global structural responses are required [20,21]. A further improvement in this direction comes from the higher order transverse shear theories (HOT) that belong, as well as CLPT and FSDT, to the so-called equivalent single layer theories (ESL). In these theories, the in-plane displacements are suitable smooth functions of the transverse coordinate, with the number of the published shear shape functions assumption being large and varied (see for instance [5,16,17,19,26,31,32]). HOT are more accurate than CLPT and FSDT, but the continuity of the shear strains at interfaces leads to a discontinuity in the shear stress distribution, and although, in principle, they do not require the use of SCF, for highly heterogeneous laminates they lead to a very stiff behavior.

The most suitable technique for the analysis of composite materials is the use of three-dimensional solid finite elements. However it becomes prohibitively expensive as the number of layers in the laminate increases (it can be as large as one hundred), in optimization analysis or for non-linear problems. It is feasible to group multiple layers within one single layer with combined properties in order to maintain the number of degrees of freedom (DOFs) of the problem within manageable limits as suggested in [18]. The accuracy in transverse shear stresses can also be improved using hybrid elements including stresses as additional DOFs [35].

Layer-wise approaches, in which the thickness of the laminate is divided into a number of layers which may or may not coincide with the physical number of layers, are more accurate than ESL theories. A through-the-thickness approximation of the displacements at layer level is assumed. A review of these techniques can be seen in [27]. These techniques have the same drawback of using three dimensional solid elements as the number of layers increases.

For angle-ply laminates and those with a low order of heterogeneity one can consider a smooth transverse shear variation across the thickness as proposed by HOT including, for instance, the trigonometric shear deformation theory (TSDT) [5,17,24,25].

For sections with a high degree of heterogeneity, the analysis with solid models and layer-wise approaches shows that the in-plane displacement profiles are far from a smooth curve that could be approximated by a polynomial of third order or higher. This has led to the so called zigzag theories where the in-plane displacement functions are only C^o continuous with a zigzag profile, possibly with strong discontinuities in the derivatives (associated to the transverse shear strain) to fit the IC of adjacent layers with shear modulus that can differ by several orders of magnitude. A review of the evolution of these theories can be seen in [2]. More recently, a refined version of this approach has been presented [29], where two hierarchical DOFs are added to the five DOFs of the FSDT enhancing the linear through-the-thickness interpolation. This approach leads to constant transverse shear stresses at each layer (i.e. discontinuous) as they are computed from the constitutive equations; however it allows dealing with clamped boundary conditions, a limitation of the previous zigzag theories in which it is based on.

This refined zigzag theory (RZT), that makes the use of SCF unnecessary, has been implemented in 2D beam finite elements [12,23,3,22], in flat plate finite elements [29,4,13,34,1] where a very good approximation to the in-plane displacements has been reported, in shell finite elements with linear kinematics [33] and with large displacements and small strains [8]. The piece-wise constant transverse shear stresses, calculated directly from the computed strains and the constitutive relations for each layer, show frequently a poor approximation. An accurate evaluation of shear stresses requires the through-the-thickness integration of the in-plane equilibrium equations, which involves *ad hoc* schemes for the computation of the derivatives of the in-plane stresses between finite elements. To avoid the *a posteriori* integration of the equilibrium conditions and to improve the predicting capabilities of the RZT, a mixed approach has been developed for beams with linear kinematics [28]. Based on the latter a so denoted RZT$^{m}_{(3,2)}$ has been developed for beams [15], where the mixed approach is combined with improvements in both the in-plane and transverse displacements interpolation. Smeared quadratic and cubic terms are added to the piece-wise linear zigzag interpolation that partially meets the stress boundary conditions at external surfaces while a quadratic interpolation of the transverse displacement is included. An extension to flat plates of the mixed approach, considering two separate states of cylindrical bending, has been presented in [14]. Such mixed approach is not yet available for general double-curved shells.

In this paper a general formulation for the mechanical analysis of composite laminated structures is proposed. The model employs solid-shell finite elements with large displacements and considers different transverse shear strains approximations in a unified way. The elements considered are a tri-linear 8-node hexahedron [9] and a 6-node triangular prism [7] in which two refined zigzag approaches (RZT), and a HOT (TSDT) are implemented. To attain this purpose the TSDT kinematic is modified and suited to work with FSDT-shell and solid-shell finite elements. In addition, this modification allows to unify the general formulation presented in this paper. The scope of this work is restricted to small elastic strains but large displacements and rotations.

An outline of this paper is as follows. Next section provides a short description of the formulation of the solid-shell elements considered. Then the additional displacement fields and the associated strains are introduced. Resulting elasticity matrices for the new generalized stress and strain measures are then evaluated. Several examples are presented in Section 5 to show the very good correlation with theoretical results, with shell models and with 3D solid discretizations. Finally some conclusions are summarized.

2. Solid-shell elements

Two prismatic solid-shell elements are considered, namely a triangular (NN=6) and a quadrilateral (NN=8) based one. The original and deformed geometries of the element are described by the standard isoparametric approximations [36].

$$\mathbf{x}(\boldsymbol{\xi}) = \sum_{I=1}^{NN} N^I(\boldsymbol{\xi})\, \mathbf{x}^I = \sum_{I=1}^{NN} N^I(\boldsymbol{\xi})\, (\mathbf{X}^I + \mathbf{u}^I) \tag{1}$$

where $\mathbf{X}^I$, $\mathbf{x}^I$ and $\mathbf{u}^I$ are the original coordinates, the present coordinates and the displacements of node I respectively. The shape functions $N^I(\boldsymbol{\xi})$ are the usual Lagrangian interpolation functions in terms of the local coordinates $\boldsymbol{\xi} = (\xi, \eta, \zeta)$ of the corresponding master element ($\boldsymbol{\xi}^I = (\xi^I, \eta^I, \zeta^I)$ are the coordinates of node I of the master element in the parametric space, see Fig. 1)

- for the 8-node brick element

$$N^I(\boldsymbol{\xi}) = \frac{1}{8}(1 + \xi\xi^I)(1 + \eta\eta^I)(1 + \zeta\zeta^I) \tag{2}$$

- for the 6-node prism element the in-plane interpolation uses area coordinates $(\xi_1, \xi_2, \xi_3) = (\xi, \eta, 1 - \xi - \eta)$ instead

$$N^I(\boldsymbol{\xi}) = \frac{1}{2}(1 - \zeta)\xi_I \quad I = 1..3 \tag{3}$$

$$N^I(\boldsymbol{\xi}) = \frac{1}{2}(1 + \zeta)\xi_{I-3} \quad I = 4..6 \tag{4}$$

Following a standard approach, at each point of interest the Cartesian derivatives are computed using the Jacobian matrix