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Crack problem in superconducting cylinder with exponential distribution of critical-current density

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ABSTRACT

The general problem of a center crack in a long cylindrical superconductor with inhomogeneous critical-current distribution is studied based on the extended Bean model for zero-field cooling (ZFC) and field cooling (FC) magnetization processes, in which the inhomogeneous parameter η is introduced for characterizing the critical-current density distribution in inhomogeneous superconductor. The effect of the inhomogeneous parameter η on both the magnetic field distribution and the variations of the normalized stress intensity factors is also obtained based on the plane strain approach and J-integral theory. The numerical results indicate that the exponential distribution of critical-current density will lead a larger trapped field inside the inhomogeneous superconductor and cause the center of the cylinder to fracture more easily. In addition, it is worth pointing out that the nonlinear field distribution is unique to the Bean model by comparing the curve shapes of the magnetization loop with homogeneous and inhomogeneous critical-current distribution.

1. Introduction

Although crack problems of high-temperature superconductors (HTSs) have been extensively studied recently, there are still many issues to be further concerned and explored. As well known, fracture behaviors have been investigated for superconductor slabs, circular cylinders or circular disks with a center-situated or inclined crack, edge crack, a central kinked crack and a penny-shaped crack, etc. [1–5]. For example, Zhou and co-workers made significant progress in addressing the properties of crack-driven fracture of a superconducting slab and cylinder [6–8]. Among these studies, the dynamic fracture behavior of the crack under electromagnetic force in bulk superconductors was investigated by Yong et al. [9–11] who considered the cases of the central crack and the edge crack. Zeng et al. [12] investigated the edge-crack problem for a long cylindrical superconductor with FEM, where the shielding effects of cracks on currents were considered. Zhang et al. [13] carried out experimental and analytical studies on the current around a through-edge crack in a cylindrical high temperature superconductor. Wang et al. [14] further investigated the inclined crack problem in a long superconductor slab. A simple model was developed by Xue et al. [15] to analyze the fracture behavior of an inclined crack interacting with a circle inclusion in a high-Tc superconducting slab based on the critical Bean model and the distribution dislocation technology. Gao et al. [16–18] analyzed the fracture behavior of the nonhomogeneous superconducting slab based on the eigenvalue and eigenvector analyses with Bean model by the methods of singular integral equations which were solved numerically by the Lobatto–Chybeshev collocation method. Furthermore, Feng et al. [19–21] also presented the multiple isoparametric finite element method (MIFEM) which was used to investigate external circumferential crack problem of a functionally graded superconducting cylinder subjected to electromagnetic forces, in which a crack reference region was defined to reflect the effects of crack on flux and current densities, and the magnetically impermeable crack surface condition and the generalized Irie–Yamafuji critical state model outside the crack region were adopted.

However, a quantitative understanding of the fracture behavior for a long circular cylindrical superconductor is still lacking. In particular, all of the studies mentioned above have been almost reported based on the homogeneous critical-current density distribution in bulk superconductors. In fact, the cracks are formed during oxygenation in the single grain bulk superconductors, in which the distribution of the critical-current density is essentially inhomogeneous due to the inhomogeneous distribution of various defects or dislocations during the preparation process of HTSs [22–27]. For example, the penetration and exit of magnetic flux

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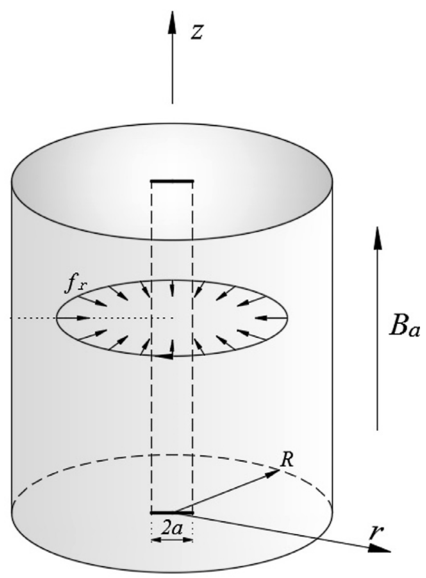

Fig. 1. Sketch of a central crack in a long cylinder superconductor under a parallel magnetic field.

in type-II superconductors were investigated by Schuster et al. [28–30] for the realistic situation, the field patterns obtained by this general theory are compared with patterns observed magneto-optically at the surface of square and rectangular of HTSs with homogeneous and inhomogeneous critical-current distribution. Subsequently, applying a larger magnetic field to a bulk superconductor containing defects may lead to the failure of such devices in practical applications. Therefore, it keeps the incentive to understand whether the spatial dependence of critical-current density makes for different effect on the mechanics properties including the fracture behaviors in the inhomogeneous superconductors.

In this paper, the fracture behavior induced by flux pinning is investigated in inhomogeneous superconductor with the exponential distribution of critical-current density. Firstly, the influence of the exponential distribution of critical-current density on the magnetic field distribution in bulk superconductor with inhomogeneous critical-current distribution is discussed in detail and compared with the homogeneous case. Then, based on the plane strain approach and J-integral theory, the dependence of the variations of the normalized SIFs on the exponential distribution of critical-current density is interpreted and illustrated graphically for ZFC and FC processes, respectively. Finally, the conclusion of this paper is presented.

2. Problem formulation

Consider a long cylindrical superconductor containing an internal central crack placed in a magnetic field B_a oriented parallel to the cylinder axis (z axis), which is illustrated in Fig. 1. The length of crack and the radius of the superconducting cylinder are respectively $2a$ and R. For the model of simplicity, this geometry is assumed to be isotropic and infinite in the z direction to neglect the demagnetization effects.

For characterizing the inhomogeneous critical-current density distribution in superconductors, it is assumed to be exponential distribution along the radius of the cylinder based on actual conditions, i.e., $J_c = \mp J_{c0} \cdot e^{\eta r}$, $\eta \geq 0.0$, where η denotes the inhomogeneous parameter with respect to the critical-current density distribution, especially when $\eta = 0.0$ it represents homogeneous critical-current distribution in the cylinder, which is shown in Fig. 2. In addition, since previous studies have shown that the in-

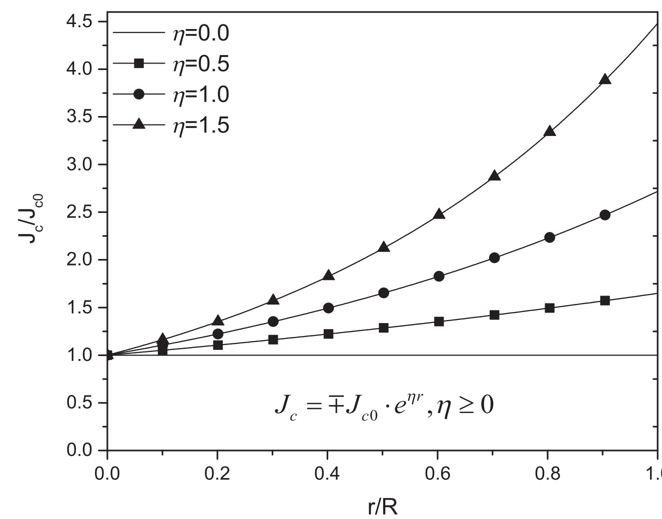

Fig. 2. Exponential distribution of critical-current density with respect to the inhomogeneous parameter η in inhomogeneous superconductor.

fluence of the closed current loops in the crack region $0 < r < a$ on the stress intensity factors (SIFs) can be neglected actually regardless of the shapes of the current loops. Thus, it is assumed that the shielding currents outside the crack region flow in circular loops and are not disturbed by the crack. For a long cylindrical superconductor, $B(r)$ can be calculated by the critical current distribution,

$$\int_r^R dr = -\frac{1}{\mu_0} \int_{B(r)}^{B_a} \frac{dB}{J_c} \tag{1}$$

Owing to the low tensile strength of HTSs, superconducting samples will be easily damaged during the applied field descent. The central crack problem of a superconducting cylinder is studied only for the decreasing field stage. For different magnetization stages, the flux profiles and crack problem induced by the electromagnetic force will be discussed for ZFC and FC magnetization processes, respectively.

2.1. ZFC process

When the applied field B_a is reduced from the maximum field B_m to 0, there are two situations decided by B_a should be considered for the ZFC process, namely, (i) the current is reversed in the outer part of the cylinder and the compressive and expansive forces are simultaneously existent, as well as the negative and positive currents outside the crack. (ii) The current is reversed in the entire cylinder and the body force becomes tensile in the entire cylinder. The parameter field B^* is denoted as the external field when the cases change from (i) to (ii), and the corresponding characteristic field B_p equals to the full penetration field of a solid cylinder without cracks, which is defined as in the extended Bean model,

$$B_p = \frac{\mu_0 J_{c0}}{\eta} (e^{\eta R} - 1) \tag{2}$$

The flux density in a quarter of the cross section is given for case (i): $B_m \geq B_a \geq B^*$ as,

$$B(r) = B_m - \frac{\mu_0 J_{c0}}{\eta} (e^{\eta R} - e^{\eta r}), 0 \leq r \leq r_0$$

$$B(r) = B_a + \frac{\mu_0 J_{c0}}{\eta} (e^{\eta R} - e^{\eta r}), r_0 < r \leq R \tag{3}$$

in which