ARTICLE IN PRESS

Engineering Failure Analysis xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Engineering Failure Analysis

journal homepage: www.elsevier.com/locate/engfailanal

The Theory of Critical Distances to assess failure strength of notched plain concrete under static and dynamic loading

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ARTICLE INFO

Keywords:
Un-reinforced concrete
Notch
Static loading
Dynamic loading
Critical distance

ABSTRACT

The Theory of Critical Distances (TCD) is a design method that is widely used in situation of practical interest to estimate the strength of notched/cracked components subjected to either static, dynamic, or fatigue loading. The TCD makes use of a characteristic length to post-process the linear-elastic stress fields damaging the material in the vicinity of the stress concentrators being designed. The employed length scale parameter depends on the specific microstructural features of the material under investigation. By making the most of the TCD's unique features, the present paper summarises an attempt of reformulating this powerful theory to make it suitable for assessing static and dynamic strength of notched plain concrete. The accuracy and reliability of the proposed reformulation of the TCD is checked against a number of experimental results that were generated by testing, under different displacement rates, square section beams of plain concrete containing notches of different sharpness. This validation exercise allowed us to demonstrate that the proposed reformulation of the TCD, which is based on the use of simple power laws, is capable of accurately assessing the static and dynamic strength of the notched un-reinforced concrete being tested, with the estimates falling within an error interval of ± 20%. The obtained level of accuracy is certainly satisfactory, especially owing to the fact that static and dynamic strength is predicted without explicitly modelling those non-linearities characterising the stress vs. strain dynamic behaviour of concrete.

1. Introduction

In situations of practical interest (such as under either blast or impact loading), concrete structures have to be designed to withstand high stress/strain rates. Having recognized this as a complex structural engineering problem, since about the middle of the last century, the international scientific community has made a tremendous effort to understand and model the mechanical/cracking behaviour of concrete materials subjected to dynamic loading. This issue has been addressed extensively by tackling this problem both from an experimental and a theoretical angle. Following the pioneering work done by Hopkinson, Davies and Kolsky [1–3] as well as Mellinger and Birkimer [4] used high velocity projectiles to strike concrete cylindrical specimens and induce spalling failure under high strain rate conditions. Since then, a number of experimental investigations (see, for instance, Refs. [5,6] and the references reported therein) have confirmed that, at room temperature, both the compressive and tensile strength of concrete tend to increase with the increase of the loading/displacement/strain rate.

After the advent of Linear Elastic Fracture Mechanics (LEFM), a few investigations were carried out also to study the existing relationship between material fracture toughness and loading rate. In particular, much experimental evidence [7,8] suggests that, at

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http://dx.doi.org/10.1016/j.engfailanal.2017.07.018
Received 20 April 2017; Received in revised form 6 July 2017; Accepted 12 July 2017

Nomenclature

a_f, b_f	material constants in the σ_f vs. $\dot{\varepsilon}$ relationship
a_K, b_K	material constants in the K_{Id} vs. $\dot{\varepsilon}$ relationship
$f_{\sigma_0}(\dot{Z})$	calibration function for $\sigma_0(\dot{Z})$.
$f_{\sigma_f}(\dot{Z})$	calibration function for $\sigma_f(\dot{Z})$
$f_{K_{Id}}(\dot{Z})$	calibration function for $K_{Id}(\dot{Z})$
r_n	notch root radius
x	generic material property
x_d	value of material property x under dynamic loading
x_s	value of material property x under quasi-static loading
DIF	Dynamic Increase Factor
K_{Ic}	plane strain fracture toughness
K_{Id}	dynamic fracture toughness
K_t	stress concentration factor
L	critical distance
Oxyz	system of coordinates
$\dot{Z}$	reference dynamic variable
α_L, β_L	material constants in the L vs. $\dot{Z}$ relationship
α_{σ_0}, β_{σ_0}	material constants in the σ_0 vs. $\dot{Z}$ relationship
$\dot{\varepsilon}$	strain rate
$\dot{\varepsilon}_s$, γ_s, α_s	reference constants in Table 1
σ_0	inherent strength
σ_1	maximum principal stress
σ_{cs}	static uniaxial compressive strength
σ_{eff}	effective stress
σ_f	failure stress
σ_{fn}	notch failure nominal stress referred to the net area
σ_{nom}	nominal stress
σ_y	normal stress parallel to axis y
σ_{UTS}	ultimate tensile strength
θ, r	polar coordinates
$\dot{\Delta}$	displacement rate

room temperature, concrete's fracture toughness can either remain constant or increase as the Stress Intensity Factor (SIF) rate increases, this mainly depending on the existing interactions between crack propagation mechanisms and material micro/meso-structural features.

Despite the large body of knowledge available to structural engineers designing concrete structures against dynamic loading, examination of the state of the art shows that a commonly accepted design approach has not yet been agreed by the international scientific community. Furthermore, the sensitivity of concrete to the presence of finite radius notches has never been investigated systematically in the past. Consequently, there are no specific approaches suitable for designing notched plain concrete against static and dynamic loading.

In this challenging scenario, by taking full advantage of the so-called Theory of Critical Distances, the present paper reports on an attempt of formulating a unifying design methodology suitable for performing static/dynamic assessment of notched plain concrete.

2. Mechanical/cracking behaviour of plain concrete under dynamic loading

Concrete is a three phase material (i.e., cement paste, aggregates, and transition zone) whose mechanical properties vary locally. When concrete is loaded dynamically, cracks are seen to propagate through those material regions characterised by higher local resistance, causing aggregate interlocking or further micro-cracking [9–11]. In contrast, under very low loading rates, the stiffness and toughness of the aggregates can lead to crack deflection, forcing the cracks themselves to grow along those paths requiring the least amount of energy for the propagation process to take place [9–12]. Furthermore, under low loading rates, large voids can arrest

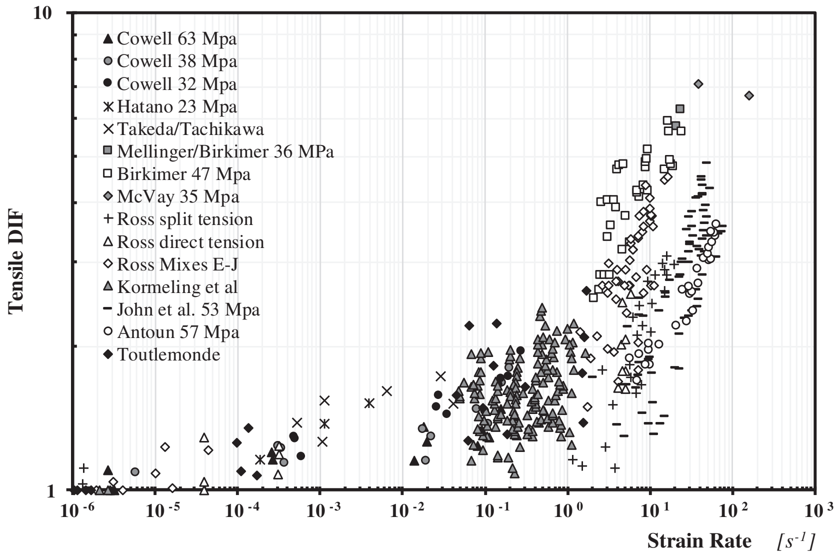

Fig. 1. Tensile DIF vs. strain rate (after Malvar & Crawford [21]).