Journal of Economic Dynamics & Control 33 (2009) 1824–1836

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Can a stochastic cusp catastrophe model explain stock market crashes?

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ARTICLE INFO

Article history:
Received 11 February 2008
Accepted 17 April 2009
Available online 12 May 2009

JEL classification:
C01
C53

Keywords:
Stochastic cusp catastrophe
Bifurcations
Singularity
Nonlinear dynamics
Stock market crash

ABSTRACT

This paper is the first attempt to fit a stochastic cusp catastrophe model to stock market data. We show that the cusp catastrophe model explains the crash of stock exchanges much better than other models. Using the data of U.S. stock markets we demonstrate that the crash of October 19, 1987, may be better explained by cusp catastrophe theory, which is not true for the crash of September 11, 2001. With the help of sentiment measures, such as the index put/call options ratio and trading volume (the former models the chartists, the latter the fundamentalists), we have found that the 1987 returns are bimodal, and the cusp catastrophe model fits these data better than alternative models. Therefore we may say that the crash has been led by internal forces. However, the causes for the crash of 2001 are external, which is also evident in much weaker presence of bifurcations in the data. In this case, alternative models explain the crash of stock exchanges better than the cusp catastrophe model.

1. Introduction

Unpredictable behavior of stock markets, especially unexpected crashes, has been a nightmare for the financial world ever since capital markets came into existence. Catastrophe theory attempts to unfold part of the information we might need to understand crash phenomena. It describes how small, continuous changes in control parameters, or independent variables influencing the state of the system, can have sudden, discontinuous effects on dependent variables. The theory is widely applicable as it can be used to describe a sudden collapse of a bridge under slowly mounting pressure, freezing of water when the temperature is gradually decreased, or the stability of black holes. In this paper, we apply the theory to sudden stock market changes that are known as crashes. Zeeman (1974) was the first to qualitatively describe the "unstable behavior of stock exchanges" by Thom (1975) catastrophe theory. We extend his ideas by incorporating a quantitative analysis in a stochastic setup.

This article provides an extension of contemporary knowledge as it puts the theory to test on financial data. As only a few papers deal with empirical applications of catastrophe theory – for instance in the field of physics, Aerts et al. (2003), Tamaki et al. (2003); chemistry, Wales (2001), biology, Torres (2001), and van Harten (2000); in the social sciences, Holyst et al. (2000); economics, Balasko (1978), Ho and Saunders (1980) or Jammemegg and Fischer (1986) – this paper contributes to that research. We build on Zeeman's qualitative description, but instead of using his model we use a

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0165-1889/$ - see front matter
doi:10.1016/j.jedc.2009.04.004

randomly perturbed version, and the primary aim of this research is to answer the question of whether stochastic catastrophe models are capable of indicating stock market crashes.

The structure of this paper will be as follows. In the first part the basic principles of catastrophe theory will be introduced. Stochastic catastrophe theory and a review of statistical testing methods will be discussed. These methods have been crucial in the history of catastrophe theory, mainly in the hands of critics like Zahler and Sussman (1977) who widely criticized catastrophe theory for non-existence of methods enabling its statistical testing. In the second part, we argue that there exists a consistent theory for statistical testing and we discuss in detail Zeeman's main hypotheses about the instability of stock markets. The role of fundamentalists and chartists and their influence on stock market changes will also be discussed.

What we regard as the most significant aspect of this paper is estimating a cusp catastrophe on real-world financial data. Our key hypothesis is that the cusp catastrophe model is able to fit the data better than an alternative linear regression model, and/or nonlinear(logistic) model. We fit the catastrophe model to the data of the October 19, 1987 crash, known as Black Monday which was the greatest single-day loss (20.5%) that Wall Street has ever suffered in continuous trading. For comparison, we use another large crash, that of September 11, 2001. The final part is devoted to the hypothesis that while in 1987 the crash was caused by internal forces, in 2001 there were external forces, namely the 9/11 terrorist attack, that caused the crash. Thus the catastrophe model should fit the data of 1987 well, as bifurcations leading to instability are present. However, it should not perform better than linear regression does on the 2001 data. As Zeeman's original model considers returns of the stock market rather than prices, we follow his analysis, and use Standard and Poor's 500 index returns as the behavioral variable. As the control variables we use the measures of sentiment, precisely the OEX[1] put/call ratio which appears to be a very good measure of speculative money (i.e. in Bates, 1991; Finucane, 1991; or Wang et al., 2006) in the capital market, against the daily change of total trading volume, the ratio of advancing stocks volume and declining stocks volume, the Dow Jones Composite Bond Index, and a one-day lag of S&P 500 returns as good proxy for large, fundamental investors.

2. Catastrophe models

Catastrophe theory has been developed by the mathematician René Thom (1975) to help explain biological morphogenesis as one of the great mysteries confronting mathematical biology. The range of potential applications is, however, extremely broad as catastrophe theory is closely related to the theory of Taylor series approximations (Cobb and Zacks, 1985). Zeeman (1974) was the first to propose its application to stock market behavior. Although his work focused on qualitative descriptions rather than quantitative applications, his hypotheses were very interesting at that time. Unfortunately, catastrophe theory had to wait until its time came, mainly due to the spreading criticism led by Zahler and Sussman (1977) and Sussman and Zahler (1978a, b). Their arguments against catastrophe theories are based on excessive reliance on qualitative methods, inappropriate quantization in some applications, use of excessively restrictive or narrow mathematical assumptions, and nonexistence of statistical theory which would enable quantitative research to be performed on real-world data. Perhaps, the discussion is also ignited by the very name of the theory, which seems rather provocative; however, it has been chosen to emphasize one of the nontrivial aspects of the behavior of nonlinear dynamic models.[2]

Although Sussman and Zahler made some good points, their criticism has initiated debates that have persisted through several decades until now. The most recent contribution has been made by Rosser (2007), who in fact ridicules the previous criticisms. He summarizes the discussion and shows that the arguments which have caused the main incomprehension are at least petty. On the other hand, nonexistence of a statistical theory was clearly a problem in that time, which has led to reliance on qualitative methods. Statistical methods have thus quickly started to be the focus of the research. Cobb (1981) and Cobb and Watson (1980) provided a reliable method for estimation of the cusp catastrophe models based on maximum likelihood estimation (MLE). Two other methods have been developed: one by Guastello (1984) who used a simple regression technique, and the least-squares estimation method of Oliva et al. (1987) GEMCAT.[3] Finally, Hartelman (1997) proposed a consistent invariant stochastic catastrophe theory for empirical verification and testing. Poston and Stewart (1978), Guastello (1987) and Rosser (2007) provided a fairly comprehensive review of the related literature, while Rosser (2007) provided a good review of those few papers applying the model to business, finance and economics.

2.1. Basic framework

A key idea in catastrophe theory is that the system under study is driven toward an equilibrium state. Wagenmakers et al. (2005a) illustrated this by imagining the movement of a ball on a curved one-dimensional surface, as in Fig. 1. The ball represents the state of the system, whereas gravity represents the driving force.

[1] OEX are options with the Standard & Poor's 100 index underlying.
[2] Cobb and Zacks (1985).
[3] A general multivariate methodology for estimating catastrophe models.