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Design and analysis of associative memories based on external inputs of continuous bidirectional associative networks

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ABSTRACT

This paper presents an extended continuous bidirectional associative memory network (CBAM) and a new ring recurrent network to behave as associative memories with external inputs. The proposed networks are robust in terms of design parameter selection. Some globally exponential stable criteria are derived for the networks with high storage capacity. The approach, by generating networks where the input data are fed via external inputs rather than initial conditions, enables multiple prototype patterns to be retrieved simultaneously. The results improve and extend some previous related works. Recurring to the numerical method, several applicable examples are given to illustrate the effectiveness of the proposed networks.

1. Introduction

Learning and memory are two highly important cognitive functions in intelligence [1]. The term associative memories refers to a brain-style system for storing prototype patterns, which can be recalled by probes with information about the content of the patterns. When given a noisy or corrupted prototype pattern (i.e., probe), the retrieval dynamics of the associative memories will converge to the prototype pattern. To date, much work has investigated associative memories via recurrent neural networks [1–4].

In the hardware circuit implementation of analog neural networks, delays inevitably occur, because of the signal transmission and the finite switching speed of amplifiers [15,16,19]. This may affect dynamical behavior and lead to an oscillation phenomenon or instability of networks [18,20]. Also, it is noted that recurrent neural networks usually have a spatial extent because of the existence of a multitude of parallel pathways with a variety of axon sizes and lengths [17]. Therefore, it is necessary to draw on distributed delays for recurrent neural networks.

In designing associative memories, there are two types of modes based on recurrent neural networks. One is to design proper weights for neural network [6–8] such that state variables are asymptotically stable at multiple equilibriums as memorized points. Singular value decomposition method [5], iterative algorithms [9] and the pseudo-inverse technique [10] were presented for the design of associative memories. These associative memories based on neural networks [6–8] are dependent on initial states and yield some spurious patterns. Therefore, prototype patterns may not be recalled accurately because each equilibrium point has an attractive region and neuron state variables of different attractive regions will converge to different equilibrium points.

To overcome this imperfection, another way was recently proposed. The idea is to design associative memory procedures, where input data are fed via external inputs and neuron state variables converge to a global equilibrium point as memorized pattern. This approach to study of neural networks stems from Grassi's work [3,4]. Grassi presented a new approach that depended only on the input not on the initial state. His seminal work has inspired other researchers [1,11,12,14] to develop networks for associative memories. In [1], stored patterns were retrievable by the proposed discrete-time recurrent neural networks with high storage capacity. In [11], two new design procedures for auto-associative and for hetero-associative memory were separately presented based on continuous neural networks with external inputs. By the second approach [1,3,4,11–14], the spurious equilibria would be avoided, and the initial states of neural networks could be given randomly.

Regarding with the design of associative memories, the previous neural networks based on external inputs consider only the case where one associative pattern can be recalled at a time. In practice, two or more patterns may be needed to recall

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simultaneously. In this case, can we solve it using external input? To explore this question, a study on recalling multiple associative pattern is necessary.

Motivated by these considerations and the works of [11,12], our objective in this paper is to investigate the associative memories based on CBAM models with delays. The main contributions of this paper are summarized as follows.

(1) The approach, by generating CBAMs where the input data are fed via external inputs rather than initial conditions, enables two prototype patterns to be retrieved each time.
(2) It is worth pointing out that our model has no limit on the selection of initial state, while it is required to be zero in [14].
(3) To recall more prototype patterns simultaneously, a new ring recurrent network is designed to behave as associative memories of multiple patterns. This ring network is viewed as the extension of neural networks [1,3,4,11–14].
(4) The proposed CBAM has higher storage capacity than those in [1,11,12], and is robust in terms of design parameter selection.
(5) Unlike the existing results of neural networks in [25,26], simulation results with real applications to binary patterns are given to demonstrate the effectiveness of CBAM.

The structure of this paper is arranged as follows. In Section 2, a CBAM model suitable for associative memory is presented. In Section 3, we demonstrate the existence and uniqueness of the equilibrium point. In Section 4, some stability criteria are established for the proposed CBAM model. In Section 5, to recall more patterns simultaneously, a new ring recurrent neural network with distribued delays is presented. In Section 6, numerical simulations illustrate the effectiveness of the proposed CBAM. Finally, Section 7 concludes this paper.

2. Preliminaries

2.1. Problem description

Write $\{-1,1\}^{p+q}$ for the set of $(p+q)$-dimensional bipolar vectors, i.e., $\{-1,1\}^{p+q}=\{X\in R^{p+q}, X=(X_1,X_2,\dots,X_{p+q})^T, X_i=1\ or-1, i=1,2,\dots,p+q\}$, denote $\{-1,1\}^{p+q}\times\{-1,1\}^{(\hat{p}+\hat{q})}$ as the product of the set of $(p+q)$-dimensional and $(\hat{p}+\hat{q})$-dimensional bipolar vectors, where $\{-1,1\}^{p+q}\times\{-1,1\}^{\hat{p}+\hat{q}}=\{(X,Y)\in R^{(p+q)\times(\hat{p}+\hat{q})}\}, X=(X_1,X_2,\dots,X_{p+q})^T, X_i=1\ \ or-1, i=1,2,\dots,p+q, Y=(Y_1,Y_2,\dots,Y_{\hat{p}+\hat{q}})^T, Y_j=1\ or-1, j=1,2,\dots,\hat{p}+\hat{q}$, $\|\cdot\|$ is the Euclidean 1-norm of R^n. The design problem is depicted as follows.

Design problem. Given $m(m<min(2^{p+q},2^{\hat{p}+\hat{q}}))$ pairs of vectors $(((^{X^{(1)}}_{Y^{(1)}}),(^{I^{(1)}}_{J^{(1)}})),$

$$\left(\begin{pmatrix}X^{(2)}\\Y^{(2)}\end{pmatrix},\begin{pmatrix}I^{(2)}\\J^{(2)}\end{pmatrix}\right),\dots,\left(\begin{pmatrix}X^{(m)}\\Y^{(m)}\end{pmatrix},\begin{pmatrix}I^{(m)}\\J^{(m)}\end{pmatrix}\right)\Bigg),$$

where $(^{I^{(l)}}_{J^{(l)}})$ is external inputs, $(^{X^{(l)}}_{Y^{(l)}})$ is stored pattern, $((^{X^{(l)}}_{Y^{(l)}}),(^{I^{(l)}}_{J^{(l)}}))\in\{-1,1\}^{\hat{p}+\hat{q}}\times\{-1,1\}^{p+q}$, $(^{X^{(l)}}_{Y^{(l)}})=(X_1^{(l)},\cdots,X_{\hat{p}}^{(l)},Y_1^{(l)},\cdots,Y_{\hat{q}}^{(l)})^T\in\{-1,1\}^{\hat{p}+\hat{q}}$, $(^{I^{(l)}}_{J^{(l)}})=(I_1^{(l)},\dots,I_p^{(l)},J_1^{(l)},\dots,J_q^{(l)})^T\in\{-1,1\}^{p+q}$, design an associative memory based on a neural network such that if external inputs $(^{I^{(l)}}_{J^{(l)}})$ is fed to the network, then the output of the network converges to corresponding stored pattern $(^{X^{(l)}}_{Y^{(l)}}), l\in\{1,2,\dots,m\}$.

2.2. A CBAM model

In this paper, a continuous-time recurrent neural networks with distributed delays is considered and the dynamics of each neuron are described by the following set of nonlinear different equations:

$$\begin{cases}\dfrac{dx_i}{dt}=-a_ix_i(t)+\displaystyle\sum_{j=1}^{q}c_{ij}\int_{\tau_0}^{\tau}K_{ij}(\theta)f_j(y_j(t-\theta))d\theta+I_i^{(l)},\\ \dfrac{dy_j}{dt}=-b_jy_j(t)+\displaystyle\sum_{i=1}^{p}d_{ji}\int_{\tau_0}^{\tau}H_{ji}(\theta)g_i(x_i(t-\theta))d\theta+J_j^{(l)},\\ X_i(t)=g(x_i(t)),\ \ t\in[0,+\infty),\\ Y_j(t)=f(y_j(t)),\ \ t\in[0,+\infty),\end{cases}\tag{1}$$

with initial values

$$\begin{cases}x_i(t)=\phi_i(t),\quad t\in[-\tau,0],\\ y_j(t)=\psi_j(t),\quad t\in[-\tau,0],\end{cases}\tag{2}$$

where $i\in P=\{1,2,\cdots,p\}$, $j\in Q=\{1,2,\dots,q\}$, $l\in\{1,2\dots,m\}$, $x_i(t)$ and $y_j(t)$ denote the potential of ith neuron and the jth neuron at time t respectively, non-negative delays τ_0 and τ correspond to finite speed of axonal signal transmission, $0\le\tau_0<\tau$, a_i and b_j stand for the self-inhibition with which the ith neuron and the jth neuron will reset their potential to the resting state when isolated from the other neurons and inputs, the connection weights c_{ij} and d_{ji} represent the strengths of synaptic connections among the circuit neurons in the network, the delay kernels $K_{ij}(\theta)$ and $H_{ji}(\theta)$ are generalized kernel functions with respect to probability distribution [24], $K_{ij}(\theta)\ge0$, $H_{ji}(\theta)\ge0$ for $\theta\ge0$ and $K_{ij}(\theta)=0, H_{ji}(\theta)=0$ when $0\le\theta<\tau_0$ or $\theta>\tau$, $I=(I_1,I_2,\dots,I_p)^T$, $J=(J_1,J_2,\dots,J_q)^T$ are input vectors, $X=(X_1,X_2,\dots,X_p)^T, Y=(Y_1,Y_2,\dots,Y_q)^T$ are output vectors, f_j and g_i are the standard activation functions with following forms illustrated in Fig. 1

$$f_j(x)=g_i(x)=\frac{1}{2}(|x+1|-|x-1|),\ \ j\in Q,\ \ i\in P,\tag{3}$$

which are employed in [1–4,11,14].

Denote the first two equations of our CBAM (1) as BAM (1).

Remark 1. It is noted that bidirectional associative memory networks (BAM) generalize the continuous Hopfield circuit model and can be obtained as a special case from the model of Cohen and Grossberg [22]. If the delay distribution functions $K_{ij}(\theta)=\delta(\theta-\tau_{ij}), H_{ji}(\theta)=\delta(\theta-\sigma_{ji})$, $\tau_0\le\tau_{ij}\le\tau$ and $\tau_0\le\sigma_{ji}\le\tau$, then $\int_{\tau_0}^{\tau}K_{ij}(\theta)f_j(y_j(t-\theta))d\theta=f_j(y_j(t-\tau_{ij}))$, $\int_{\tau_0}^{\tau}H_{ji}(\theta)g_i(x_i(t-\theta))d\theta=g_i(x_i(t-\sigma_{ji}))$, then BAM (1) becomes a bidirectional associative network with discrete delays considered in [25,27]; If $\tau_0=0$ and $\tau=+\infty$, then BAM (1) is similar to bidirectional associative with distributed delays investigated in [26]. Therefore, our BAM (1) can contain previous bidirectional associative neural networks [25–27].

For further investigating the associative memories based on CBAM (1), we introduce the following Definitions.

Definition 1. CBAM (1) is an auto-associative memory when $I^{(l)}=X^{(l)}, J^{(l)}=Y^{(l)}, l=1,\dots,m$.

Definition 2. The equilibrium point $(x^{*T},y^{*T})^T$ of neural network defined by (1) is said to be globally exponentially stable if there exist constants $M>0$ and $\lambda>0$, such that the output trajectory of this network satisfies

$$\left\|\begin{pmatrix}x(t;\varphi)-x^*\\y(t;\psi)-y^*\end{pmatrix}\right\|\le\left\|\begin{pmatrix}\varphi(t)-x^*\\\psi(t)-y^*\end{pmatrix}\right\|_{\tau}Me^{-\lambda t},\ \ for\ all\ t\ge0,$$

where

$$\|(^{\varphi(t)-x^*}_{\psi(t)-y^*})\|_{\tau}=\sup_{-\tau\le t\le0}\|(^{\varphi(t)-x^*}_{\psi(t)-y^*})\|.$$

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