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Growth of symbolic number knowledge accelerates after children understand cardinality

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ABSTRACT

Children who achieve an early understanding of the cardinal value of number words (cardinal knowledge) have a superior understanding of the relations among numerals at school entry, controlling other factors (e.g., intelligence). We tested the hypothesis that this pattern emerges because an understanding of cardinal value jump starts children's learning of the relations among numerals. Across two years of preschool, the cardinal knowledge of 179 children (85 boys) was assessed four times, as was their understanding of the relative quantity of Arabic numerals and competence at discriminating nonsymbolic quantities. Children were more accurate on nonsymbolic than numeral comparisons before they understood cardinality, but showed more rapid growth for numeral than nonsymbolic comparisons once they understood cardinality. Moreover, and with the possible exception of very small numerals (< 5), before they understood cardinality children were no better than chance in their numeral comparisons, but greatly exceeded chance once they understood cardinality. These patterns were independent of the age at which children became cardinal principle knowers and independent of intelligence, executive function, and preliteracy skills. More broadly, the results provide a developmental bridge between cardinal knowledge and school-entry number knowledge.

1. Introduction

Children's understanding of the relations among numerals and their ability to use arithmetical procedures to manipulate these relations – hereafter, number system knowledge –are foundational to their long-term mathematical development (Geary, 1994; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Siegler & Braithwaite, 2017). And, just as critically, the extent of this development by school completion influences employability, wages, and on-the-job productivity, above and beyond the influence of intelligence, reading ability, and demographics (e.g., Bynner, 1997; Rivera-Batiz, 1992). At this point, we do not fully understand all of the factors that contribute to children's long-term mathematical development (Bailey, Duncan, Watts, Clements, & Sarama, 2018; Bailey, Watts, Littlefield, & Geary, 2014), but nevertheless this development is per force composed of domain-specific knowledge and skills and one of the factors that contributes to the accretion of this knowledge is prior, more basic knowledge within the domain (Geary, Nicholas, Li, & Sun, 2017). The age at which children acquire these basic skills may be important to their long-term development, but the influence of age-of-acquisition is not well understood. A child who acquires a basic skill six months earlier than other children has in effect six additional months to build onto this knowledge. The cumulative nature of mathematics and the expectation that children's domain-specific mathematical knowledge will be continuously upgraded from one academic year to the next may make any such age-of-acquisition effects particularly important.

We recently found such an effect, whereby children who understood cardinality (the quantity represented by number symbols, below) by 4 years-of-age had more elaborated number system knowledge at the beginning of 1st grade relative to their peers who achieved this conceptual insight a year or so later (Geary et al., 2018). We hypothesized that children's learning of the relations among Arabic numerals begins only after they understand cardinality, and thus children who achieved this insight at a younger age had more time to elaborate their number system knowledge than did their peers. In this follow-up study, we more directly test this hypothesis by examining growth in children's knowledge of the relative magnitudes of Arabic numerals before and after they understand the cardinality principle, controlling domain-general abilities.

The hypothesis seems straightforward and perhaps not in need of empirical confirmation, but the state of the field is such that the links between early number knowledge and number knowledge at school

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entry are vigorously debated (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2017), and could benefit from empirical bridges between these two critical junctures in children's mathematical development. To provide a contrast and an egress into the heart of current debate regarding the basic foundations of mathematical development, we also examine the relation between cardinal knowledge and growth in children's ability to discriminate between nonsymbolic quantities (Shusterman, Slusser, Halberda, & Odic, 2016).

2. Early number knowledge

Given the importance of mathematical competencies for educational and job opportunities in adulthood and for navigating the quantitative complexities of daily life (e.g., Reyna, Nelson, Han, & Dieckmann, 2009), identifying the initial foundation upon which these competencies are built is of theoretical and practical importance. Indeed, considerable effort has been expended toward this end during the past decade and has led to the identification of two candidates for this foundation. The first is the approximate number system (ANS), an evolutionarily ancient system that enables the comparison of the relative quantities of collections of objects (Feigenson, Dehaene, & Spelke, 2004; Geary, Berch, & Mann Koepke, 2015). Young children who have a finely tuned ANS have higher concurrent and prospective mathematics achievement than their peers with a less sensitive ANS (e.g., Bonny & Lourenco, 2013; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013). Starr and colleagues found that 6-month-olds' ANS acuity predicted their mathematics achievement and knowledge of the cardinality of number words three years later. And, several meta-analyses have confirmed a small ($r \sim 0.2$–0.3) but reliable relation between measures of ANS acuity and mathematics achievement (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2017).

At the same time, the proposition that the ANS is the key foundational support for later mathematics achievement has been questioned (Carey, Shusterman, Haward, & Distefano, 2017; De Smedt, Noël, Gilmore, & Ansari, 2013). The alternative and second foundational candidate is children's understanding of the cardinal value or magnitude of number symbols, and their understanding of the relations among them (Bugden & Ansari, 2011; Chu, vanMarle, & Geary, 2015; De Smedt & Gilmore, 2011; Fazio et al., 2014; Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007). As reviewed by De Smedt et al. (2013), the most commonly used measure of this symbolic knowledge is children's speed and accuracy of comparing the relative magnitudes of two Arabic numerals (see also Schneider et al., 2017). The distance effect – speed and accuracy of comparisons vary systematically with the quantitative difference between the compared numerals – is also a commonly used measure and reflects the integration of numerical magnitudes into a number-knowledge network. The gist of these studies is that the fluency and eventually the automaticity of processing the quantitative relations among numerals is an important component of children's competence with symbolic mathematics.

Stated somewhat differently, children who are fluent in numeral comparisons at school entry appear to have advantages in subsequent mathematics learning. It takes many years to build fluency (Holloway & Ansari, 2009), and thus children who can correctly order numerals early in the preschool school years have more time to build fluency before school entry than their later developing peers. Children of course must first understand the cardinal value represented by number symbols (number words and numerals) before they understand the relations among two or more of them (Carey et al., 2017; Knudsen, Fischer, Henning, & Aschersleben, 2015; cf. Gunderson, Spaepen, & Levine, 2015; Le Corre, 2014). Our point is that the age at which children understand cardinality is an inflection point in their mathematical development and this inflection occurs in part because it is a precursor to their understanding of the quantities represented by numerals and the latter is a precursor to their building fluency in numeral comparisons.

The first step on the path to cardinal knowledge in turn occurs when children learn by rote the first few number words and the count list ("one, two, three…"; Fuson, 1988; Gelman & Gallistel, 1978). These first steps are typically taken between 2- and 3-years of age, and over the next two years are followed by a gradual understanding of the magnitudes represented by number words (Carey, 2004; Le Corre & Carey, 2007; Spelke, 2017; Wynn, 1992). The latter is well understood and involves learning, one-by-one, the number of objects associated with the first few number words. One knowers understand that 'one' refers to one and only one object of any kind, and two knowers understand the quantities represented by 'one' and 'two'. Sometime after children understand 'three' or 'four', they have the conceptual insight that all number words refer to specific quantities and that each successive number word in their count list represents exactly one more than the word before it (Le Corre & Carey, 2007; Sarnecka & Carey, 2008). Whether the latter results from the insight that every successive number represents $n + 1$ or that counting can be systematically used to generate cardinality is debated (Cheung, Rubenson, & Barner, 2017; Le Corre & Carey, 2007; Wynn, 1990). Either way, these children are considered cardinal principle knowers (CPK).

During this timeframe, children are also learning to name Arabic numerals and are learning the quantities represented by them, although their learning of numeral names and their associated magnitudes is a step or two behind their learning of number words (Knudsen et al., 2015). For instance, a 4-year-old who knows the cardinality of 'one' to 'four', inclusive, would typically know the cardinal values of the numerals '1', '2', and perhaps '3'. The gap between number word and numeral knowledge closes substantially between 4- and 5-years and completely, at least for small values, by 6 years (Knudsen et al., 2015). Here, the key point is that while preschool children are learning the names and magnitudes of numerals they are also beginning to accurately compare their relative magnitudes. Preschoolers' competence with numeral comparisons in turn directly links the early development of numeral knowledge to the core measures on which the symbolic foundation hypothesis of mathematical development has been based (e.g., De Smedt et al., 2013).

The central questions here are whether the rate of learning the relations between the quantities represented by numerals accelerates, once children become cardinal principle knowers. And, whether this rate of change differs from across-age improvement in ANS acuity (Halberda & Feigenson, 2008). The latter contrast assesses whether cardinal principle knowers gain more in symbolic than nonsymbolic quantity judgments after they achieve CPK status. The contrast does not rule out a relation between the ANS and early symbolic learning, and in fact we have found such a relation (vanMarle et al., 2018), and Shusterman et al. (2016) found that performance on ANS acuity tasks improves after children's achieve CPK status. Rather, we are interested in the more specific relation between children's cardinal principle status and the timing of this status on their nascent knowledge of the relations among numerals, knowledge that appears to be the first step toward building fluency with numeral comparisons.

3. Current study

If our hypothesis that children's knowledge of the relations among numerals is dependent on their understanding of cardinality is correct, then their accuracy in comparing two numerals should not differ from chance before they become cardinal principle knowers and improve rapidly after they become cardinal principle knowers. As a contrast, their performance on measures of ANS acuity should be above chance before children become cardinal principle knowers, although their performance may also improve once they understand cardinality (Shusterman et al., 2016). In any case, the gap between performance on these symbolic and nonsymbolic measures of children's understanding