Knowledge-Based Systems 142 (2018) 71–84

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

A Dynamic Three-way Decision Model based on the Updating of Attribute Values

Qinghua Zhang[a,b,*], Gongxun Lv[a], Yuhong Chen[a], Guoyin Wang[a]

[a] Chongqing Key Laboratory of Computational Intelligence, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[b] College of Computer Science and Technology, Chongqing University of Posts and Telecommunications, Chongqing 400065 China

ARTICLE INFO

Article history:
Received 17 May 2017
Revised 16 November 2017
Accepted 22 November 2017
Available online 24 November 2017

Keywords:
Three-way decision
Updating of attribute values
Dynamic decision-making
Feature extraction
Thresholds

ABSTRACT

The three-way decision model is a topic of substantial research interest in the field of artificial intelligence, and many researchers have focused on to its feasibility and rationality. The tolerance and practicability of the three-way decision model are better than those of the two-way decision model. When the attribute value of each object in a domain is given, the formation of a three-way classification of the domain is a key issue. However, few studies have been conducted on establishing a three-way decision model with the given attribute values in the case where the number of objects in an accepted region is given. Therefore, in the model presented in this paper, both the uncertainty of attribute values and the cost of updating are fully considered. In this paper, first, a new concept of attribute ratio is defined to describe an object when the attribute value of the object is numerical, and then, a dynamic three-way decision model is established. Second, a feature extraction algorithm of attribute values is proposed, and a pair of decision thresholds of the dynamic three-way decision model is also obtained according to the given conditions. Then, in the case where the attribute values are updated, an example is provided to demonstrate how two-way classification results can be obtained in the dynamic decision-making process. Finally, the results of simulation experiments show that the proposed model is feasible and effective in practical applications. When the number of objects in an accepted region has been given, according to the updating strategy of attribute values, the three-way decision problems are successfully solved by the proposed model.

1. Introduction

The three-way decision model [1] is a new method for handling the uncertainty problem that has emerged in recent years. It is a “trisecting and acting” type of decision-making model evolved from the decision-theoretic rough sets model, which accords with human cognition habits [2]. The core content of three-way decisions is that a domain is divided into three disjoint regions and the corresponding decision-making strategies for each region are obtained. The three-way decision model, as an important extension of the traditional two-way decision model, takes many uncertainty factors in the decision-making process into account. The deferred decision [1] is viewed as the third decision-making behavior when the information is not sufficient to determine the state of an object, that is, whether the object is accepted or rejected. The three-way decision model has been widely used in medical diagnosis [3], social judgment theory [4], statistical management [5], manuscript review [6], face recognition [7], and other fields. In recent years, many scholars studied the transformation of a naive decision theory into a theoretical system, information processing model, or calculation method [8]. Significant progress has been made in the three-way decision model and its applications, with the development of decision analysis [9–12], three-way clustering analysis [13,14], filter spam email [15,16], three-way decisions space [17], cost-sensitive three-way decisions [18–20], three-way decisions and game-theory [21–23], multi-granulation three-way decisions [24–26], sequential three-way decisions [27–29], dynamic three-way decisions [30–32], three-way formal concept analysis [33], three-way decisions and domain-logic [34], etc.

Many research studies on three-way decision theory [35–50] were based on the corresponding background of decision making and its applications. For example, Li and Zhou [51] proposed a new decision model based on decision-theoretic rough sets combined with the different risk preferences of decision makers. Yang and Yao [23] presented a multi-agent three-way decision model that describes how to develop a comprehensive and consensus decision-making standard when the different users give differ-

* Corresponding author.
E-mail addresses: 445365223@qq.com (Q. Zhang), Lvgx410@163.com (G. Lv).

https://doi.org/10.1016/j.knosys.2017.11.026

ent decision sets according to their own standards. Zhou and Li [52] suggested a method of multi-level decision rule extraction based on decision-theoretic rough sets. Liu et al. [53–55] studied both the three-way decisions-theoretic rough set models of multi-classification and the three-way decision method based on discriminant analysis.

However, for the case where the number of objects in an accepted region is given, few models and theories for forming three regions have thus far been proposed in the research field. Therefore, based on dynamic decision making with the updating of attribute values, a three-way decision model is proposed in this paper to solve this problem. Then, a practical example is provided to describe it more comprehensively.

In the production process of handicrafts, suppose a worker has t opportunities to improve the quality of the handicrafts. According to the final score from high to low, the top k objects need to be selected from N handicrafts as the best batch ($N > k$). After all the handicrafts have been updated t times, the "optimal" classification results (an object set is divided into accepted and rejected regions directly after all the objects have been updated) can be obtained. Obviously, when N is too large, the cost of updating in this "ideal" decision-making process is excessive. Therefore, in order to decrease the cost, the selection is usually based on some artificial means of subjectively selecting some objects as the "optimal" classification results directly before the objects are updated. This selection method is a typical two-way decision model, which depends on human experience to select the top k. Obviously, this method is too direct and arbitrary. On the objective background of updating, the reason is that Rules (1) and (2) for updating exist, as follows.

(1) The lower (higher) the attribute value of the objects, the lower (higher) is its increment.
(2) In the process of updating, an "inferior" object cannot easily become an "excellent" object.

Therefore, the two-way decision model essentially ignores the uncertain factors of the actual situation. To resolve these shortcomings, a dynamic three-way decision model needs to be established, which improve the situation.

The contributions and innovation of this paper are as follows.

(1) In a set of objects with numerical attributes, when the objects cannot be classified according to the attributes, the influence degree of each object on the "excellent" distribution, that is, a new concept of optimal deviation, is defined. Then, a new extraction method of this feature(optimal deviation) is proposed based on the mathematical characteristics of the attribute values.
(2) When the number of objects in an accepted region is given and the attribute values of the objects are updated, a dynamic three-way decision model is established, which can constantly modify the classification results according to the updating strategy of the attribute values.
(3) A method for calculating adaptive thresholds is proposed such that the dynamic three-way decision model can handle the practical decision-making updating problem. The proposed model can not only achieve better classification results than the two-way decision model, but also effectively decrease the cost of updating.

This paper is organized as follows. In the second section, the basic concepts of rough sets theory are briefly reviewed. In the third section, a three-way decision model based on the updating of attribute values is established. In the fourth section, in the case where the attribute values are updated, an example is given to demonstrate the process of obtaining two-way classification results. In the fifth section, the algorithms are described, as well as the simulation experiments to validate the rationality and feasibility of the proposed model. The sixth section is the conclusion.

2. Preliminaries

Rough sets theory, proposed by Pawlak in 1982 [56,57], is a mathematical tool for dealing with uncertain knowledge. The most important feature of this theory is that it requires no prior knowledge and can extract rules from the data directly. Therefore, rough sets theory is widely applied in many fields, such as pattern recognition, machine learning, and expert systems [58,59]. In order to clarify the idea of this paper, first many basic concepts are reviewed in this section.

Definition 1. (Decision information system [56,56]) A decision information system can be expressed as

$$S = (U, AT, V, f),$$

where $U = \{x_1, x_2, ..., x_n\}$ is a finite nonempty set of objects, called the decision domain, and $AT = C \cup D$ is a finite nonempty set of attributes, called the condition domain. The subsets C and D are called the condition attributes set and decision attributes set, respectively, and $C \cap D = \emptyset$. V_a is a nonempty set of values for an attribute $a \in AT$. f: $U \times AT \to V$ is a complete information function that represents each value of each instance, where each object is given an information value. An attribute value for object x on attribute a is given, denoted by $a(x)$.

Definition 2. (Equivalence relation [56,56]) Given a subset of the attribute set $A \subseteq C$, the equivalence relation $IND(A)$ on the universe U can be defined as

$$IND(A) = \left\{(x, y) \in U^2 | \forall a \in A, a(x) = a(y)\right\}.$$

Definition 3. (Equivalence class [60]) Let $IND(A)$ denote an equivalence relation on U; then, a partition of U can be induced by $IND(A)$ and denoted by $U/IND(A)$. For a given object $x \in U$, $[x]_A$ denotes an equivalence class of x with respect to A; that is,

$$[x]_A = \{y \in U | (x, y) \in IND(A)\}.$$

For simplicity, $[x]_A$ is usually denoted by $[x]$.

Definition 4. (Upper and lower approximation sets [61]) Suppose an information system $S = (U, AT, V, f)$, for a subset $X \subseteq U$, its lower and upper approximation sets are defined respectively as

$$\underline{apr}(X) = \{x \in U | [x] \subseteq X\} \quad and$$

$$\overline{apr}(X) = \{x \in U | [x] \cap X \neq \emptyset\},$$

where $[x]$ denotes the equivalence class of x.

The family of all equivalence classes is also known as the quotient set of U; it is denoted by $U/R = \{[x]_R | x \in U\}$. The universe can be divided into three disjoint regions, namely, the positive, boundary, and negative regions:

$$POS(X) = \underline{apr}(X),$$

$$BND(X) = \overline{apr}(X) - \underline{apr}(X), \quad and$$

$$NEG(X) = U - \overline{apr}(X).$$

When elements are classified into the positive region, this means that they certainly belong to the target concept X, when they are classified into the negative region this means that they certainly do not belong to the target concept X, and when they are classified into the boundary region this means that they may belong to the target concept X. Such a model embodies the basic idea of the three-way decisions model. However, Pawlak's rough