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# Asset price dynamics in a financial market with heterogeneous trading strategies and time delays

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**Abstract**

In this paper we present a continuous time dynamical model of heterogeneous agents interacting in a financial market where transactions are cleared by a market maker. The market is composed of fundamentalist, trend following and contrarian agents who process market information with different time delays. Each class of investors is characterized by path dependent risk aversion. We also allow for the possibility of evolutionary switching between trend following and contrarian strategies. We find that the system shows periodic, quasi-periodic and chaotic dynamics as well as synchronization between technical traders. Furthermore, the model is able to generate time series of returns that exhibit statistical properties similar to those of the S&P 500 index, which is characterized by excess kurtosis, volatility clustering and long memory.


*Keywords:* Dynamic asset pricing; Heterogeneous agents; Complex dynamics; Chaos; Stock market dynamics

## 1. Introduction

In recent years there has been a growing disaffection with the standard paradigm of efficient markets and rational expectations. In an efficient market, asset prices are the outcome of the trading of rational agents, who forecast the expected price by exploiting all the available information and know that other traders are rational. This implies that prices must equal the fundamental values and therefore changes in prices are only caused by changes in the fundamental. In real markets, however, traders have different information on traded assets and process information differently, therefore the assumption of homogeneous rational traders may not be appropriate. The efficient market hypothesis motivates the use of random walk increments in financial time series modeling: if news about fundamentals are normally distributed, the returns on an asset will be normal as well. However, the random walk assumption does not allow the replication of some stylized facts of real financial markets, such as volatility clustering, excess kurtosis, autocorrelation in square and absolute returns,

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bubbles and crashes. Recently a large number of models that take into account heterogeneity in financial markets have been proposed. Contributions to this literature include [1–6]. Ref. [7] analyzes a market composed of a continuum of fundamentalists who show delays in information processing. These models allow for the formation of speculative bubbles, which may be triggered by news about fundamentals and reinforced by technical trading. Since different investors interact with one another in a nonlinear fashion, these models are capable of generating stable equilibria, periodic, quasi-periodic dynamics and strange attractors. This paper builds on the model of Ref. [7], which is inspired by the models of thermodynamics of Refs. [8–10] and analyzes a financial market only composed of fundamentalist investors who trade according to the mispricing of the asset with delays uniformly distributed from initial to current time. We generalize [7] by introducing a continuum of technical traders who behave as either trend followers or contrarians and a switching rule between these technical trading rules. We will analyze how the interaction of different types of investors with path dependent risk aversions determines the dynamics and the statistical properties of the system as key parameters are changed.

## 2. The model

Let us consider a security continuously traded at price $P(t)$. Assume that this security is in fixed supply, so that the price is only driven by excess demand. Let us assume that the excess demand $D(t)$ is a function of the current price and the fundamental value $F(t)$. A market maker takes a long position whenever the excess demand is negative and a short position whenever the excess demand is positive so as to clear the market. The market maker adjusts the price in the direction of the excess demand with speed equal to $\lambda^M$. The instantaneous rate of return is:

$$r(t) \equiv \frac{\dot{P}(t)}{P(t)} = \lambda^M D(P(t), F(t)), \quad \lambda^M > 0. \tag{1}$$

The fundamental value is assumed to grow at a constant rate $g$, therefore,

$$\frac{\dot{F}(t)}{F(t)} = g. \tag{2}$$

The market is composed of an infinite number of investors, who choose among three different investment strategies. Let us assume that a fraction $\alpha$ of investors follows a fundamentalist strategy and a fraction $(1-\alpha)$ follows a technical analysis strategy. The fraction of technical analysts is in turn composed of a fraction $\beta$ of trend followers and a fraction $(1-\beta)$ of contrarians. Let $D^F(t)$, $D^{TF}(t)$ and $D^C(t)$ be, respectively, the demands of fundamentalists, trend followers and contrarians rescaled by the proportions of agents who trade according to a given strategy. The excess demand for the security is thus given by

$$D(t) = \alpha D^F(t) + (1-\alpha)[\beta D^{TF}(t) + (1-\beta) D^C(t)], \quad \alpha, \beta \varepsilon [0, 1]. \tag{3}$$

Each trader operates with a delay equal to $\tau$, that is, the demand of a particular trader at time $t$ depends on her decision variable at time $t-\tau$. Time delays are uniformly distributed in the interval $[0, t]$. Fundamentalists react to differences between price and fundamental value. The demand of fundamentalists operating with delay $\tau$ is

$$D^{F\tau}(t) = \lambda^{F\tau} \log\left[\frac{F(t-\tau)}{P(t-\tau)}\right], \quad \lambda^{F\tau} > 0, \tag{4}$$

where $\lambda^{F\tau}$ is a parameter that measures the speed of reaction of fundamentalist traders; we will assume that $\lambda^{F\tau} = \lambda^F$ throughout the paper. This demand function implies that the fundamentalists believe that the price tends to the fundamental value in the long run and reacts to the percentage mispricing of the asset in symmetric way with respect to underpricing and overpricing. If time delays are uniformly distributed, the market demand of fundamentalists is given by

$$D^F(t) = \lambda^F \int_0^t \log\left[\frac{F(t-\tau)}{P(t-\tau)}\right] \mathrm{d}\tau, \quad \lambda^F > 0, \tag{5}$$