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Passive motion in dynamical disorder as a model for stock market prices

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Abstract

A new model for stock price fluctuations is proposed, based upon an analogy with the motion of tracers in Gaussian random fields, as used in turbulent dispersion models and in studies of transport in dynamically disordered media. Analytical and numerical results for this model in a special limiting case of a single-scale field show characteristics similar to those found in empirical studies of stock market data. Specifically, short-term returns have a non-Gaussian distribution, with super-diffusive volatility. Assuming a power-law decay of the time correlation of the disorder, the returns correlation decays rapidly but the correlation function of the absolute returns exhibits a slow power-law decay. The returns distribution converges towards Gaussian over long times. Some important characteristics of empirical data are not, however, reproduced by the model, notably the scaling of tails of the cumulative distribution function of returns. Implied volatilities for options pricing are found by numerical simulation.

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1. Introduction

Random fluctuations in stock market prices have long fascinated investors and mathematical modelers alike. Although the investors' hopes of accurately predicting tomorrow's share price appear to be in vain, models which limit themselves to statistical characteristics such as distributions and correlations have had some success. Bachelier's classical model [1] treats the stock price $S(t)$ as a random walk, leading to the conclusion that the distribution of prices is Gaussian. Samuelson [2] instead describes the log-price

$$x(t) = \ln[S(t)/S(0)] \tag{1}$$

as a random walk, and therefore concludes that the stock price $S(t)$ should have a log-normal distribution. This model remains in common usage, despite the shortcomings listed below, not least because it permits the derivation of an equation for pricing options and other financial derivatives, e.g., the famous Black–Scholes equation [3]. Nevertheless, empirical evidence from stock market data indicates that the random walk model inadequately describes many important features of the stock price process. The following stylized facts are accepted as established by these studies [3,4]:

- (i) Short-term returns are non-Gaussian, with 'fat tails' and high central peaks. The center of the returns distribution is well fitted by Lévy distributions [5]. Recent studies indicate that the tails of the returns distribution decay as power-laws [6]. As the lag time increases, the returns distributions slowly converge towards Gaussian; recent analysis of the Standard & Poor 500 index (S&P500) estimates this convergence is seen only on lags longer than 4 days [6].
- (ii) The correlation function of returns decays exponentially over a short timescale, consistent with market efficiency. However, the correlation function of the *absolute value* of the returns shows a much slower (power-law) decay [6].
- (iii) The volatility (standard deviation of returns) grows like that of a diffusion (random walk) process, i.e., as the square root of the lag time, for lag times longer than about 10 min. However, higher frequency (shorter lag) returns demonstrate a super-diffusive volatility, which can be fitted to power-laws with exponents found to range between 0.67 and 0.77 [6,7].
- (iv) The distribution of stock price returns exhibits a simple scaling: in Ref. [5] a power-law scaling of the peak of the returns distribution $P(0)$ with lag time is shown to hold across many magnitudes of lag times. The exponent of the power-law is approximately -0.7.

Most of the references cited here examine data from the Standard & Poor 500 index (S&P500), but other international markets are found to behave similarly [6].