Constellation design with geometric and probabilistic shaping

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A B S T R A C T

A systematic study, including theory, simulation and experiments, is carried out to review the generalized pairwise optimization algorithm for designing optimized constellation. In order to verify its effectiveness, the algorithm is applied in three testing cases: 2-dimensional 8 quadrature amplitude modulation (QAM), 4-dimensional set-partitioning QAM, and probabilistic-shaped (PS) 32QAM. The results suggest that geometric shaping can work together with PS to further bridge the gap toward the Shannon limit.

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1. Introduction

Advanced modulation formats have been received more and more attention due to the limited received signal-to-noise ratio (SNR) and fiber nonlinearity. Thanks to the high-speed digital coherent receiver and digital-to-analog converters (DAC), well-established multi-level quadrature amplitude modulation (QAM) formats in the wireless communication, such as 8QAM and 16QAM, are applied in the coherent optical communication to deliver beyond 100 Gbps per wavelength data rate [1]. Despite their simple implementation complexity, the recent study shows that these modulation formats are operating far away from the Shannon limit [2]. In other words, the current optical systems are not fully utilizing the capacity provided by the received SNRs.

The pursuit of capacity-approaching modulation formats is under intensive research to close the gap toward the Shannon limit. Although there are many varieties of modulation formats proposed in the literature, they can be generally divided into geometric shaping (GS) and probabilistic shaping (PS) categories. Using matrix rotation of an 8-dimensional (8-dim) bi-orthogonal format, 8-dim 2QAM has been demonstrated to outperforming its two-dimensional (2-dim) counterparts [3], such as dual-polarization binary phase-shifted keying (BPSK) and dual-polarization quadrature phase-shifted keying (QPSK). The recently proposed four-dimensional (4-dim) 8QAM [4] and two-amplitude (2A) 8PSK [5] can beat their 2-dim star-8QAM counterparts by ~0.5 dB at the forward error correction (FEC) limit. Moreover, six-dimensional 16QAM has been proposed to outperform star-8QAM by increased Euclidean distance [6]; however, its performance advantage at the FEC limit comes at the cost of a computationally expensive iterative decoding scheme. The tailoring of modulation formats has been found to become critical in accomplishing the goal to approach the fundamental capacity limit.

Although the shaping gain increases with constellation size, it is capped at 1.53 dB [7]. GS-based constellation, such as amplitude-phase-shifted keying (APSK) and 32QAM, have been demonstrated over trans-Atlantic distance at > 8 b/s/Hz SE over C-band only [8]. To further reduce the gap toward the Shannon limit, PS-based 64QAM have been demonstrated to outperforming 32QAM over 6600 km [9,10] at the expense of increased complexity non-uniform constellation shaping [11]. In [12] GS-optimized 32QAM is designed in a nonlinear optical channel and demonstrated to having higher capacity limits than 32QAM over 11 185 km transmission.

In this paper, we review the generalized pairwise optimization algorithm [13] to design GS multi-dimension constellation together with PS based on two objective functions: minimize the bit error rate (BER) or maximize the modulation capacity at the given SNR and bits mapping. The design procedure with uniform and non-uniform signaling is presented in details and the designed optimized constellation is examined in both analysis and experiments to demonstrate the effectiveness of the proposed algorithm.

The paper is structured as follows: in Section 2 the generalized pairwise optimization algorithm is reviewed together with two objective functions. In Section 3, three design cases are presented using the proposed algorithm: 2-dim 8QAM, 4-dim SP32-QAM. Section 4 summarizes the work.

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2. Pairwise optimization algorithm

Optimal 2-dim constellation was designed based on pairwise optimization algorithm at a non-uniform distribution. Two essential steps are proposed in the original paper [14]:

1. Given the target SNR, symbol error rate (SER) is first minimized by optimizing the geometric positions of a pair of symbols \((s_i, s_j)\) at a time. The optimal 2-dim constellation is derived by repeating the process after looping through all the \(M \times (M - 1)/2\) pairs for any \(M\)-QAM constellation.

2. Optimum bits mapping is exhaustively searched through layers to minimize the BER of the derived optimal geometric constellation.

In this paper, the aforementioned two steps are consolidated into one simple objective function: minimize the BER or \(-1 \times \text{GMI} \) given the bits mapping and SNR. Both objective functions can be calculated using Monte Carlo (MC) simulation. However, instead of using time-consuming MC simulation for BER computation, analytic BER equations are derived here to facilitate the optimization process.

2.1. Analytic BER of N-dimensional constellation

Considering an \(N\)-dimensional (\(N\)-dim) constellation with \(M\) non-equiprobable symbols \([s_1, s_2, \ldots, s_M]\) at probability mass function (PMF) \(p_i, i \in \{1, 2, \ldots, M\}\) to encode \(n = -\sum_{i=1}^{M} p_i \log_2 p_i\) bits, their bits mappings are represented by \(\beta_i, i \in \{1, 2, \ldots, M\}\). Note that \(s_i\)'s are \(N\times1\) vectors which are denoted in lower case bold, and scalar variables are denoted in lower case normal font. The PMF can be equiprobable, i.e., \(p_i = \frac{1}{\sqrt{N}}\), to represent uniform constellation signaling. The Hamming distance \(h(\beta_i, \beta_j)\) is defined as the number of bits encoded between symbols \(s_i\) and \(s_j\) are different.

The analytic SER between any pair of symbols \((s_i, s_j)\) in an additive white Gaussian noise (AWGN) channel can be given as [14]

\[
P_{e}(c_{ij}) = \left\{ \begin{array}{ll} 
\frac{\sqrt{4\sigma^2_n/N \ln \frac{p_j}{p_i}}}{2\|s_i - s_j\|} + \frac{\sqrt{4\sigma^2_n/N \ln \frac{p_j}{p_i}}}{2\|s_i - s_j\|}, & \text{if } i = j \\
\frac{\sqrt{4\sigma^2_n/N \ln \frac{p_j}{p_i}}}{2\|s_i - s_j\|}, & \text{if } i \neq j 
\end{array} \right.
\]

where \(Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy\) is the Gaussian Q-function, \(\| \cdot \|\) is the norm operation of a vector, and \(\sigma^2_n\) is the noise variance accumulated in \(N\)-dim space. Note that \(P_{e}(c_{ij})\) is the probability that \(s_i\) is decoded as the sent symbol given that \(s_j\) is sent.

As a result, the SER can be tightly upper bounded by

\[
P_e = \sum_{j=1}^{M} \sum_{i \neq j} P_e(c_{ij}) P(s_j) \leq \sum_{j=1}^{M} \sum_{i \neq j} \lambda_j P_e(c_{ij}) P(s_j),
\]

(2)

The Hamming distance \(h(\beta_i, \beta_j)\) between each symbol pair is taken into account when converting the SER Eq. (2) to BER:

\[
P_b \leq \sum_{j=1}^{M} P_b(\beta_i, \beta_j) P(e_{ij}) P(s_j).
\]

(3)

The derived BER upper bound from Eq. (3) is compared in Fig. 1 with the ones from MC simulation for both uniform and non-uniform 16QAM. The non-uniform 16QAM is produced by adjusting the PMF \(p_i\)'s governed by Maxwell–Boltzmann distribution depending on the symbol power:

\[
p_i = \frac{K(\lambda)}{\sum_{j=1}^{M} K(\lambda)} \exp(-\lambda \|s_i\|^2).
\]

(4)

where \(K(\lambda) = \frac{1}{\sum_{j=1}^{M} \exp(-2\lambda \|s_j\|^2)}\).

As shown in Fig. 1, the analytic BER of Eq. (3) follows closely with the ones from MC simulation for both PMF scenarios up to the BER level of \(2 \times 10^{-2}\), which is close to the state-of-the-art soft-decision FEC threshold. Moreover, the performance difference between two formats remains at high BER regime despite they drift away from the MC curves. The density plots of these two formats are plotted in the right insets of Fig. 1.

Due to the high accuracy of the analytic BER equations derived here, the pairwise optimization process can take advantage of fast computation of analytic BER to design the optimal geometric shaping within a few seconds.

2.2. GMI calculation of N-dim constellation

In a bit-interleaved coded modulation (BICM) system with a suboptimally mismatched bit-wise decoder, generalized mutual information (GMI) is regarded as an effective approach to compare its post-FEC capacity of a signal constellation \(X\) encoding \(n\) bits per symbol [15]. Its channel capacity can be represented by BICM-GMI:

\[
C_{\text{GMI}} = H(X) - \sum_{i=1}^{M} H(B_i | Y),
\]

(5)

where \(H(X)\) denotes the entropy of signal \(X\) associated with a PMF \(p(a_i), a_i \in X\), given by \(H(X) = -\sum_{i=1}^{M} p(x_i) \log_2 p(x_i)\). The conditional entropy \(H(B_i | Y)\) computes uncertainty between the random binary variables \(B_i \in \{0, 1\}\) at \(i\)th position and the channel output \(Y\). Note that GMI derived here is not the upper bound of the BICM capacity under bit-wise decoder, and its upper bound analysis is beyond the scope of this paper. Despite that GMI can be numerically derived as proposed in [15], MC simulation is used in our paper to estimate the GMI by generating \(N\) random symbols from constellation \(X\) following a specified PMF \(p(a_i)\).

The received complex symbols are given by \(y = x + n\), where \(n\) is complex AWGN with zero mean and variance \(\sigma^2_n\). At the given SNR \(E[\|s_i\|^2]/\sigma_n^2\), its BICM-GMI capacity can be computed as

\[
C_{\text{BICM}} = -\mathbb{E}[\log_2 p(a_i)] + \mathbb{E} \left[ \sum_{i=1}^{M} \log_2 \frac{\sum_{j=1}^{M} \sum_{a_j} \rho(y | a_j, a_i) p(a_i) p(a_j)}{\sum_{a_j} \rho(y | a_j, a_i) p(a_i)} \right],
\]

(6)

where \(\mathbb{E}[\cdot]\) and \(p(y | a_i)\) represent the expectation operation and conditional probability of the received channel outputs given the input symbol \(a_i\), \(a_n^i\) and \(x_n^i\) denote the transmitted symbols with bit \(b \in \{0, 1\}\) at \(i\)th position, \(\|x_n^i\|\) is an indicator function that equals to 1 when \(x_n^i = b\) otherwise 0. In the MC simulation, Eq. (6) can be approximated by

\[
C_{\text{GMI}} = H(X) + \mathbb{E} \left[ \log_2 \frac{h(j) - L_1(j) + (1 - h(j)) \cdot L_0(j)}{P_j(j)} \right],
\]

(7)

where \(h(j) \in \{0, 1\}\) denotes the transmitter bits for the \(j\)th symbol at \(i\)th bit and \(P_j(j)\) is the probability of the received \(j\)th symbol \(y(j)\). Note that the \(\mathbb{E}[\cdot]\) in Eq. (7) is performed with respect to the symbol index \(j\), \(L_0\) and \(L_1\) stand for the bit likelihood (\(L\)) for the received \(j\)th symbol \(y(j)\), which can be computed based on the following equation:

\[
L_{b=0,1} = \sum_{x_n^i \in \chi^i} p(y | x_n^i, b=0,1) p(x_n^i, b=0,1) = \frac{\sum_{x_n^i \in \chi^i} \{2 \pi \sigma_n^2\}^{-n/2} \exp \left(-\frac{\|y - x_n^i, b=0,1\|^2}{2\sigma_n^2} \right)}{p(x_n^i, b=0,1)},
\]

(8)

where \(\sigma^2 = \sigma_n^2/N\) is the noise variance in each dimension. Note that \(\sigma^2\) can be estimated from the recovered constellation in the experiments.

GMI is shown to be affected by both bits mapping and geometric shapes of the constellation. It is well-known that non-Gray-mapped constellation, such as 8QAM, 32QAM and other multi-dimensional constellation, suffers from a BICM-capacity loss because of smaller GMI is shown to be affected by both bits mapping and geometric shapes of the constellation. It is well-known that non-Gray-mapped constellation, such as 8QAM, 32QAM and other multi-dimensional constellation, suffers from a BICM-capacity loss because of smaller GMI is shown to be affected by both bits mapping and geometric shapes of the constellation. It is well-known that non-Gray-mapped constellation, such as 8QAM, 32QAM and other multi-dimensional constellation, suffers from a BICM-capacity loss because of smaller GMI is shown to be affected by both bits mapping and geometric shapes of the constellation. It is well-known that non-Gray-mapped constellation, such as 8QAM, 32QAM and other multi-dimensional constellation, suffers from a BICM-capacity loss because of smaller
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