Computers and Structures 183 (2017) 1–13

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Adjoint design sensitivity analysis and optimization of nonlinear structures using geometrical mapping approach

Yintao Wei [a,*], Chonglei Zhao [a], Zhenhan Yao [b], Patrice Hauret [c], Xuebing Li [a], Michael Kaliske [d]

[a] State Key Lab Automotive Safety & Energy, Tsinghua University, Beijing 100084, China
[b] Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China
[c] Mécanique Numérique - CTEU/PM/SIM/ER, Centre de Technologies Michelin, 63040 Clermont-Ferrand, Cedex 9, France
[d] Institute for Structural Analysis, Technische Universität Dresden, Dresden, Germany

ARTICLE INFO

Article history:
Received 20 February 2016
Accepted 12 January 2017

Keywords:
Shape optimization
Adjoint variable method
Nonlinear structures with contact forces
Sensitivity analysis
Geometrical mapping approach

ABSTRACT

A new adjoint shape design sensitivity formulation for nonlinear structures subject to contact forces is developed. The method is based on a geometrical mapping approach where shape variation is regarded as a mapping characterized by the shape variation velocity field. An adjoint variable method is developed for performing sensitivity analysis of the average shear strain in the tire belt area, which profile is properly parameterized in function of arcs, allowing explicit design velocities calculated. It can be seen that the present method is much faster than the direct differentiation method and more accurate than the classical finite difference scheme.

1. Introduction

Sensitivity analysis in shape optimization problems is complicated by the fact that the profile of the structure may change sharply in the optimization process [1]. This issue becomes even more critical in highly nonlinear problems such as, for example, optimal design of automotive tires whose mechanical response, durability and dynamical forces/moments generated in the contact patch strongly depends on the outer profile of the structure. Shape design sensitivity analysis (DSA) is still a challenge for tire industry researchers due to the high level of nonlinearity in material, geometry and contact stresses [2]. In this case, finite element analysis obtains an optimal design at affordable computational cost.

Global optimization algorithms that combine different types of response surface approximations are more efficient than local search methods that may get trapped in local minima. The Kriging method is more robust than standard response surface fitting because it does not require assuming the form of the response surface a priori. The gradient-based Kriging (GBK) method [3] can further reduce the number of samples. However, the issue of disposing of a stable and accurate sensitivity formulation becomes even more critical in this case [3,4]. Another way to reduce the computational cost of shape optimization is to use sample points determined by means of the Latin hypercube design minimizing pairwise correlations and maximizing inter-site distances of sample points [4,5].

The material derivative approach (MDA) (see the textbook by Choi and Kim [6,7]) has been adopted in different engineering areas by [8–20]. The adjoint variable method (AVM) [8,21–34] allows improving computational efficiency of sensitivity evaluations. However, the theoretical derivation of AVM for shape DSA of highly nonlinear structures as tires is a challenging task in view of the complexity of kinematic relationships. Here, a new adjoint shape DSA formulation is proposed for nonlinear tire structures with contact forces. The new formulation entails two basic aspects: (i) derivation of design velocity using a geometrical mapping approach (GMA) and (ii) formulation of shape DSA for nonlinear tire structures starting from the parameterized tire profile. The final goal is to perform accurate and stable DSA at low computational cost.

The idea behind GMA is that shape variations are regarded as a mapping characterized by the shape variation velocity field. Using the virtual work principle, sensitivity equations of state variables (i.e., displacements) with respect to the shape variation are formulated. The AVM for sensitivity analysis of average shear strain in the tire belt area is hence formulated. The tire profile is parameterized using parameterized arcs, which makes boundary design velocities explicitly calculable according to its design variables. A passenger radial tire is chosen as case study: remarkably, AVM is

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* Corresponding author.
E-mail address: weiyt@tsinghua.edu.cn (Y. Wei).

http://dx.doi.org/10.1016/j.compstruc.2017.01.004
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about 80% faster than direct differentiation method (DDM) in this case. This provides a useful framework for shape sensitivity analysis and optimization of highly nonlinear structures.

The paper is structured as follows. Section 2 describes design variables and objective function of the tire shape optimization problem. Section 3 presents the geometrical mapping approach that will be used for deriving sensitivity formulation. Section 4 illustrates how tire shape is parameterized and presents the definition of shape design velocity. Section 5 explains how to formulate sensitivity of displacements and displacement gradients with respect to shape variables. Sensitivity derivation for the objective function considered in this study is presented in Sections 6 and 7, respectively for the DDM and AVM. Applications to a passenger radial tire contour optimization are discussed in Section 8. Finally, Section 9 summarizes the main findings of this study.

2. Shape variables and objective function

This article addresses nonlinear structures with contact forces, with tires being the application example. The tire shape optimization problem in this study included the five design variables shown in Fig. 1: radius of the first crown arc R_1, radius of the second crown arc R_2, radius of the shoulder arc R_3, radius of the sidewall-shoulder arc R_4, meridian coordinate of the intersection point of the first and second arcs W_1, tread contact width W_2. Since R_4 is not an independent variable as its value can be known from W_2, the radius related to the shape of the inner boundary and parameters remain constant in the optimization.

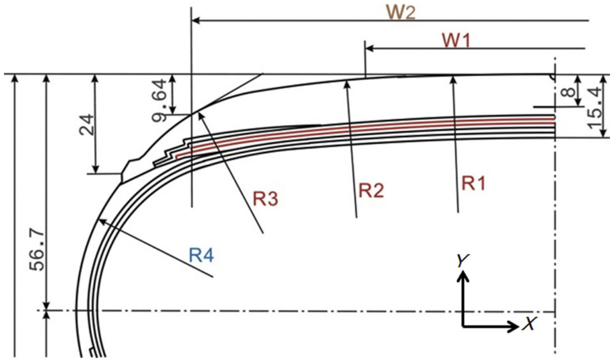

Fig. 1. Design variables in the shape optimization of a tire (units/mm).

The objective function to be minimized is the average effective shear strain which develops in the belt end area near the tire-ground contact region. The corresponding area in the meridian plane is shown in Fig. 2, and the 3D finite element model of tire is obtained from this 2D model by rotating around the center of tire. The circle of 360 degrees is divided into 50 sections on average and the footprint length of tire tread in the circumferential direction comes to 6 sections. Therefore the circumferential angle is taken as $2\pi \times 6/50$ which is shown in Fig. 4. The objective function (also called performance function) can be expressed as

$$ {}^t_\tau\psi^1 = \frac{1}{{}_\tau V_\mathrm{b}} \int_{{}_\tau V_\mathrm{b}} {}^t_\tau\bar{\varepsilon}(\boldsymbol{x})\mathrm{d}_\tau V \tag{1} $$

where the left subscript τ denotes the initial configuration corresponding to the shape parameter τ_α ($\alpha = 1, 2\ldots5$) mentioned in Fig. 1, the left superscript t denotes the load parameter, ${}_\tau V_b$ is the total volume of the design area, and ${}^t_\tau\bar{\varepsilon}(\boldsymbol{x})$ [35] is the effective shear strain:

$$ {}^t_\tau\bar{\varepsilon} = \sqrt{\frac{2}{3}{}^t_\tau e_{ij}\,{}^t_\tau e_{ij}} = \sqrt{\frac{2}{3}\left({}^t_\tau\varepsilon_{ij} - \frac{1}{3}{}^t_\tau\varepsilon_{kk}\delta_{ij}\right)\left({}^t_\tau\varepsilon_{ij} - \frac{1}{3}{}^t_\tau\varepsilon_{ll}\delta_{ij}\right)} \tag{2} $$

Fig. 2 shows the FE mesh adopted for the meridian plane of the tire. The model is comprised of different rubber materials and rebars elements as reinforcements. The FE model, homogeneously divided into 50 sections in the circumferential direction, includes in most part trilinear hexahedral elements (HEX8). The level of mesh refinement was such to have mesh independent solutions at reasonably low computational cost. The structure is subject to two independent load cases: (i) inflation pressure and (ii) contact with rigid road surface under the load carried.

3. Geometrical mapping approach

In order to evaluate the sensitivity of cost function with respect to shape variations, strain sensitivities must be determined. For that purpose, sensitivity of displacements with respect to changes in shape must be obtained first. In this study, the GMA was utilized because the MDA may be misleading as the real material derivative is not suitable for formulating the variation in shape design. The partial derivative with respect to the shape parameter does not have any physical meaning, and the gradient is the derivative with respect to the Euler coordinates rather than to the Lagrange coordinates that appear in the displacement gradient of the solids. The GMA is presented with clear physical meaning, wherein the shape variation is regarded as a mapping characterized by the shape vari-

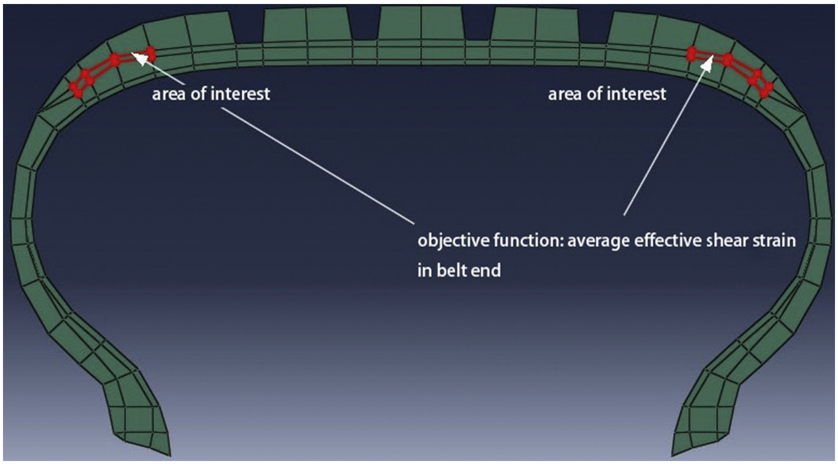

Fig. 2. FE model of the meridian plane of the tire and definition of the cost function in belt end area.