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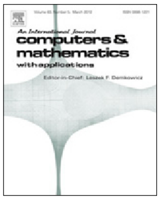

From geometric design to numerical analysis: A direct approach using the Finite Cell Method on Constructive Solid Geometry

Benjamin Wassermann[a,*], Stefan Kollmannsberger[a], Tino Bog[a], Ernst Rank[a,b]

[a] *Chair for Computation in Engineering, Technische Universität München, Arcisstr. 21, 80333 München, Germany*
[b] *Institute for Advanced Study, Technische Universität München, Lichtenbergstr. 2a, 85748 Garching, Germany*

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ABSTRACT

During the last ten years, increasing efforts were made to improve and simplify the process from Computer Aided Design (CAD) modeling to a numerical simulation. It has been shown that the transition from one model to another, i.e. the meshing, is a bottleneck. Several approaches have been developed to overcome this time-consuming step, e.g. Isogeometric Analysis (IGA), which applies the shape functions used for the geometry description (typically B-Splines and NURBS) directly to the numerical analysis. In contrast to IGA, which deals with boundary represented models (B-Rep), our approach focuses on parametric volumetric models such as Constructive Solid Geometries (CSG). These models have several advantages, as their geometry description is inherently watertight and they provide a description of the models' interior. To be able to use the explicit mathematical description of these models, we employ the Finite Cell Method (FCM). Herein, the only necessary input is a reliable statement whether an (integration-) point lies inside or outside of the geometric model. This paper mainly discusses such point-in-membership tests on various geometric objects like sweeps and lofts, as well as several geometric operations such as filleting or chamfering. We demonstrate that, based on the information of the construction method of these objects, the point-in-membership-test can be carried out efficiently and robustly.

1. Introduction

Computer aided engineering in general requires an iterative process to find an optimal design. This iterative process consists of a modeling phase followed by a numerical simulation and an analysis phase.

Modern CAD tools mainly use two different techniques to create 3D models. A classic method, which is still commonly used, is boundary representation (B-Rep) [1]. B-Rep describes a body implicitly as a topological model via its faces, edges, and nodes. Geometric information is then assigned to faces and edges, often using B-Spline-, or NURBS surfaces and curves. A more recent and natural approach is Procedural Modeling (PM), which is strongly related to Constructive Solid Geometry (CSG), but extends this concept by providing additional operations and primitives. Both CSG and PM describe a complex model as a combination of simple or complex primitives and Boolean operations (union, intersection, difference).

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[*] Corresponding author.

E-mail addresses: benjamin.wassermann@tum.de (B. Wassermann), stefan.kollmannsberger@tum.de (S. Kollmannsberger), tino.bog@tum.de (T. Bog), ernst.rank@tum.de (E. Rank).

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Procedural modeling and B-Rep each have advantages and disadvantages, which are often complementary in such a way that, nowadays, many CAD systems use a hybrid representation combining B-Rep and PM [2]. In this context, the B-Rep model provides information necessary e.g. for visualization purposes. PM serves as an underlying model that can easily be used for parametric and feature-based design [3], for which a description of the construction history, the dependencies, and the constraints are mandatory. It is noteworthy that it is always possible to derive a B-Rep model from a PM model, but not the other way round. This is due to the loss of information in the conversion from PM to B-Rep. In addition, B-Rep cannot provide information about the structure of the interior of the model. However, this information can be crucial, for example in cases of heterogeneous materials or to describe additive manufacturing processes. Interestingly, fully three-dimensional-computational mechanical analyses mostly draw the geometrical information of the computational domain from B-Rep models which are then explicitly converted into a volumetric description by a meshing process. Moreover, in the finite element method (FEM), elements are required to conform with the physical boundaries of the model, which often requires a flawless B-Rep description. A practical consequence of these requirements is the often huge engineering effort to 'clean' a CAD model or to 'heal' a finite element mesh before a numerical analysis can start. At Sandia National Laboratories [4], an estimation of the relative time required for a representative design process showed that more than 80% of the engineering effort is allotted to the transition from geometric models to simulation models that are suitable for analysis.

Various methodologies have been developed to overcome the difficulties involved in this transition process. The most prominent method in the Computational Mechanics Community is the recently introduced Isogeometric Analysis (IGA) as proposed by Hughes et al. [5]. IGA aims at bridging the gap between the CAD model and computational analysis by a closer mathematical interconnection between the two worlds. To this end, the same B-Splines and NURBS representations used to describe CAD are applied as both geometry and Ansatz functions in FEM. These functions offer several desirable properties such as the possibility of straightforward refinements in grid size and polynomial degree, as well as the possibility to control the continuity within a patch. Most importantly, they guarantee a precise description of the geometry, in contrast to classical FEM, where only an approximation can be obtained by meshing into tetrahedra or hexahedra. Furthermore, as B-Splines and NURBS are functions of higher order, they offer the potential to deliver high convergence rates if the underlying problem possesses smooth solutions. Concerning the modeling processes, IGA was first applied to B-Reps which consisted of several conforming two-dimensional B-Splines or NURBS patches. More complicated topologies are usually generated by trimming, which may lead to non-watertight geometric models. Remedies for this problem range from classic re-parametrization [6] to the use of T-Splines [7].

An alternative, designed to overcome the problems of B-Rep descriptions, are V-Reps, recently proposed by Gershon et al. [8]. They consist of trimmed trivariate NURBS patches which directly describe the volume under consideration. A related approach was presented by Zuo et al. [9], who proposed to treat CSG primitives separately as volumes using IGA and to trim and glue them by using the Mortar Method [10] at their intersection surface. However, apart from some special numerical pitfalls inherent to domain sewing techniques, this poses the additional difficulty that an explicit boundary representation needs to be set up *for all* inter-subdomain boundaries. Another related approach was presented much earlier by Natekar et al. [11] who proposed to combine spline-based element formulations with two-dimensional CSG model descriptions. However, this approach is also based on heavy use of explicit boundary representations as well as a decomposition into sub-domains. The same holds also for the design-through analysis procedure presented in [12], which uses a B-Rep description and relies on a 3D ray-casting test to describe the volume of the model.

This strong reliance on the explicit description of coupling interfaces – or, more generally, surface descriptions in the analysis process – poses a drawback to parametric modeling approaches: Even though a change of parameters or constraints hardly has any impact on the general structure of the CSG model itself, it often triggers a complete reconstruction of the entire corresponding B-Rep model. Together with the observation that a CSG or a procedural model is intrinsically watertight and directly provides information about the interior, we conclude that a desirable simulation technique would have to use the explicit description of volumes by CSG as often as possible, and its B-Rep representation as little as possible.

To this end, we propose a combination of CSG and the Finite Cell Method for volume orientated modeling and numerical analysis. We denote this approach as a direct modeling-to-analysis method as it allows, like IGA, a very close interaction of the (geometric) design process and the (numerical) analysis, where an engineer can immediately investigate consequences of a variation of the geometric design on the mechanical behavior of a structural object.

The Finite Cell Method (FCM) [13], which represents the core of this approach, is a high-order fictitious domain method that embeds an arbitrary complex geometry into an extended domain which can easily be meshed by a Cartesian grid. The complexity of the geometry is handled only on the level of integration of element matrices and load vectors. This makes the method very flexible, because the only information the FCM needs from the CAD model is a reliable and robust point-in-membership test, i.e. whether an integration point lies inside or outside of the physical model. This point-in-membership test is directly provided by the CSG model description. The interplay between CSG and FCM was already investigated for simple primitives, and it proved to be an accurate and efficient method to analyze trimmed NURBS patch structures [14]. The goal of the present paper is to extend the combination of the FCM and the CSG to more complex geometric models as well as to solid construction processes of industrial relevance.

This paper is organized as follows: In Section 2, a short overview on geometric representations and the Finite Cell Method is given. In Section 3, the relevant methods for the combination of CSG and FCM are presented. Section 4 provides examples showing the relevance and potential for practical applications before conclusions are drawn in Section 5.

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