



# Generalised non-dimensional multi-parametric involute spur gear design model considering manufacturability and geometrical compatibility



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## ABSTRACT

A generalised non-dimensional multi-parametric model for involute spur gear design is presented, considering manufacturability and geometrical compatibility, where the latter considers various modes of interference and accounts for the combined effects of the module, pressure angle, tooth addendum, dedendum, cutter tip radius, and the numbers of teeth of a pair of mating gears. The effect of the same parameters together with tooth thickness on the manufacturability of the individual gear teeth is also modelled in terms of pointing and undercutting. The full range of parameter values, including non-standard ones, is considered. The resulting combined model serves to provide a complete analytical overview of the multi-parametric design space and is suitable for the fast assessment of existing designs, for implicit or explicit (direct) gear design, for extracting design guidelines, and for design optimisation. The model can be used to identify and explore highly promising under-used subspaces of the parametric design space, which are currently of significant interest to i.e. the automotive and aerospace industries.

## 1. Introduction

### 1.1. General framing of the problem

In almost all kinds of industrial applications for motion and power transmission such as automotive, aerospace, robotics, machinery etc., gears - and involute gears in particular- are indispensable and mission critical elements of the design. Gear design itself is a complex process that, in spite of much accumulated knowledge and supporting standards, computational models [1–7] and software [8–12], still critically depends on the experience of the designer, especially if time constraints prohibit an exhaustive iterative multi-parametric search. In fact, the possible independent design parameters that must be studied even in the case of a ‘simple’ spur gear configuration comprise i.e. module, addendum, profile shift, tooth thickness, dedendum, cutter tip radius, pressure angle, face width etc, thus already numbering 8 parameters. The study is further complicated by the possibility to adjust each parameter for each gear separately. More crucially, it is important to consider the choice of these parameters in the early stage of the design, and in full consideration of the couplings (and constraints) that govern their feasible choices. The part of the design space that is not excluded because of aforementioned constraints can then be subject to parameter selection and possibly a design

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Nomenclature	
<i>Gear description &amp; indices</i>	
$\alpha_0$	pressure angle
$a_{12}$	nominal centre distance
$\alpha_c$	operating pressure angle
$\alpha_0''$	pressure angle at tip of tooth
$b$	tooth width
$c_c(c_{ci})$	cutter tip radius coefficient (of gear i)
$c_f(c_{fi})$	dedendum coefficient (of gear i)
$c_k(c_{ki})$	addendum coefficient (of gear i)
$c_s(c_{si})$	tooth thickness coefficient (of gear i)
$c_m(c_{mi})$	profile shift coefficient (of gear i)
$c_{sg}(c_{sgi})$	thickness coefficients at the base circles (of gear i)
$r_s$	form circle radius
$\hat{t}$	trochoid tip thickness of the generating rack tooth (sharp tooth)
$m$	module
$\overrightarrow{O_i R}$	vector from centre of gear i to intersection point
$A_1^{(n)} A_1^{(n+1)}$	vector from two consecutive points on the tooth profile geometry of the gear number one
$R$	intersection point of tooth profiles
$\vec{t}_i(r_i)$	tooth flank vector equation of gear no. i
$i_{1,2}$	gear transmission ratio
$O$	centre of reference coordinate system
$\theta_i$	angular position (of gear i)
$d\theta_i$	angular velocity (of gear i)
$r_b(r_{bi})$	base circle radius (of gear i)
$r_o(r_{oi})$	radius of pitch circle (of gear i)
$r_g(r_{gi})$	involute base radius (of gear i)
$r_k(r_{ki})$	outside radius
$r_f(r_{fi})$	inside radius (of gear i)
$\varepsilon$	contact ratio
$t_b$	base pitch (ref. to involute)
$t_R$	circular pitch
*	(as subscript) ref. to non-dimensional gear
$i$	ref. to gear no. i
$x$	addendum modification coefficient
$N_i$	number of teeth (of gear i)
$A_i$	point on profile of gear i
$\overrightarrow{O_i O_j}$	vector from the centre of gear i to the centre of gear j
$\overrightarrow{O_i K_j}$	vector from the centre of gear i to the tip point of gear j
$O_i$	centre of gear i
$\xi_1$	unitary vector of $\overrightarrow{A_1^{(n)} A_1^{(n+1)}}$
$\xi_2$	unitary vector of $\overrightarrow{O_i K_j}$
$K_i$	single ref. point at the corner of gear no. i
$\varphi_{si}$	angle corresponding to the tooth pitch thickness of gear i
$\vartheta$	Poisson's ratio
$\rho_i$	radii of curvature at the point of contact of gear i

optimisation study. Already, in the automotive and aerospace industry every major company is developing and exploring its own class of non-standard designs and continuous improvements are being sought to achieve higher compactness, higher strength and load carrying capacity, improved dynamics with reduced noise and vibration signatures, in particular with regard to performance under partial or reversing loads, whining, rattling etc.

Several but limited studies and supporting models exist for this kind of design space exploration: Dedicated studies exist i.e. for the influence of module [7,13], addendum [14–22], profile modification [23–29] tooth thickness [30–36], dedendum [37–42], cutter tip radius [43–46], pressure angle [47–54], etc, but without considering the simultaneous manipulation of other design parameters as well, typically assigning standard values to the latter. Additionally, many researchers have investigated the influences of each design parameter for different applications such as contact analysis [55–63], stress [64–71], vibration [72–78], dynamics [79–88], cracking [89–93], lubrication [94–101], etc, but with similar limitations to their scope of the design space. Furthermore, the knowledge present in industrial publications/ technical reports is on the other hand very specific to given machine applications and not easily generalizable or verifiable.

Hereunder we provide some selective examples of the aforementioned limitations in scope, which, while justified or necessary in the context of the respective studies, mean that the possibility to generalise the results without additional modelling is likewise limited:

### 1.1.1.1. Dedicated parametric studies

- **Module:** E.g., with regard to the influence of the module, Nonaka et al [13] studied the strength of spur gear teeth with small modules of the order of 0.1 to 1.0 mm by using a specially designed gear test rig. The failure mode of gear teeth was observed for both tempered and soft nitrided gears. A sign of uneven contact along face width was found only for very small gears of modules 0.1 and 0.2 mm, which was very unlikely for such high precision in alignments of gear axes. Influences of the choice of pressure angle or tooth proportions were not considered, limiting the scope of the study.
- **Addendum:** With regard to the effect of addendum, e.g. Li [15] investigated its influence on the tooth surface contact strength and the bending strength of spur gears. Finite element analysis, the method of mathematical programming and teeth contact model were utilized to lead the contact analyses of the loaded teeth, calculation of stress and deformation of spur gears with not the same contact ratios and addendums. Load-sharing rate, root bending stress, contact stress, mesh stiffness and also the transmission error of the spur gears were analysed. It was detected that the number of contact teeth can be increased with increasing the amount of addendum, in addition, the contact & root bending stresses can be reduced with this kind of increasing. The effects of cutter tip radius (hence fillet radius), pressure angle etc parameters were not accounted for in this study.
- **Profile modification:** E.g. Lin et al [28] investigated the influence of parabolic and linear profile modification on low contact ratio of spur gears tooth with regard to the dynamic response. The influence of the modification zone's length and the whole

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