Optik 148 (2017) 275–282

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

The caustic structure near a grazing point in the plane

Alain Joets

Laboratoire de Physique des Solides, Université Paris-Sud, CNRS, UMR 8502, F-91405 Orsay Cedex, France

ARTICLE INFO

Article history:
Received 11 May 2017
Accepted 28 August 2017

Keywords:
Caustic
Grazing point
Multiple reflections

ABSTRACT

A grazing point of a concave mirror illuminated by a large beam of rays is a point of the mirror's edge where the incident ray is parallel to the mirror. In the neighborhood of such point the rays are reflected a great number of times. We show that an ordered series of caustics passes through the grazing point, each caustic corresponding to a fixed number of reflections by the mirror. We study, in the framework of planar geometrical optics, the structure of this remarkable set of caustics. Our main result is a formula giving the curvature of the caustic curves at the grazing point as a function of the number of reflections. This sequence is universal in the sense that it is independent of the shape of the incident wavefront. A grazing point in the plane is an unstable point and we show how the caustic structure is modified under the effect of a small perturbation of the optical system.

1. Introduction

Multiple reflections on a reflector produce rich and complex configurations of rays, of wavefronts and of caustics [1–3]. In particular, we have shown in a previous paper that multiple reflections by a parabolic mirror in the plane lead to the formation of a new type of singularity, namely a *grazing point* [3]. By definition, it is a point G of the mirror M for which the incident ray is tangent to M. In other words, the angle of incidence and the angle of reflection are both equal to $\pi/2$ at a grazing point. Of course the mirror has to be concave and not convex, in order to produce multiple ray reflections. Moreover, the grazing point is necessarily located at the edge of the mirror. The set of the rays reflected exactly n times on the mirror does have generically an envelope, its caustic K_n. We succeeded in obtaining, in our case of a parabolic mirror illuminated by a beam of parallel rays, global parametrizations for the first three caustics K_1, K_2 and K_3 [3]. These parametrizations show that they pass through the grazing point, tangentially to the mirror (see Fig. 1). They also show that K_2 is located between K_1 and the mirror, whereas K_3 is located between K_2 and the mirror. These results suggest the existence of a remarkable caustic structure, ordered according to the number of reflections n, each caustic K_n being located between the caustic K_{n-1} and the mirror.

Since we are interested in an infinite set of caustic curves, it seems to be hopeless to attempt a direct determination of global parametrizations of the caustics, as we did in our previous paper for the first three caustics [3]. In fact, we are not interested in the global behavior of the caustic curves, but only in their local behavior around the grazing point. This behavior may be expressed by expanding the various quantities according to a small parameter, for instance the distance to the grazing point.

So our problem is reduced to finding the leading terms for the local parametrizations of the caustics K_n near the grazing point. On the other hand, it is clear that the law of reflection yields a relation linking the rays reflected n times to the rays

E-mail address: alain.joets@u-psud.fr

http://dx.doi.org/10.1016/j.ijleo.2017.08.137

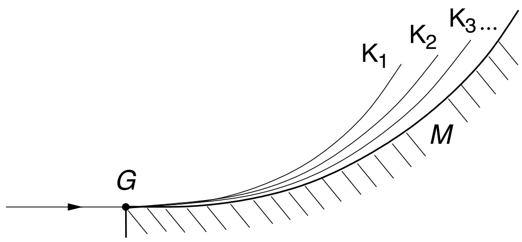

Fig. 1. By definition, a grazing point is a point G at which the incident ray (arrow) is tangent to the mirror M. The sketch shows that the caustics K_n, pass through G, tangentially to M at G.

reflected $n+1$ times. It is then expected that there exists some relation linking the leading terms of K_n to those of K_{n+1}. The main part of the present paper will be devoted to making explicit this recurrence relation with the hope that it can be solved with respect to the parameter n.

Moreover, our problem being essentially local, we may relax some conditions that were useful in our paper about the caustics associated to multiple reflections by a parabolic mirror [3]. In particular:

- we relax the condition on the parabolic shape of the mirror: the mirror's shape is now assumed to be arbitrary (but concave);
- we relax the condition on the parallelism of the incident rays: we consider an arbitrary beam of incident rays (possessing a grazing point).

As far as we know, the problem of the caustic structure near a grazing has not yet been considered. We will solve it in all its generality, i.e. not for a special type of mirror and a special type of congruence of incident rays. The framework of this study is geometrical optics in a plane. The medium is assumed to be isotropic and homogeneous. The rays propagate in straight lines until they meet the mirror where they are deviated by reflection.

2. The optical system

We consider, in the plane, a beam of incident light rays, reflected by a mirror M. We assume that there exists a grazing point G, i.e. a point on the mirror where the incident ray is tangent to M (see Fig. 1). The origin of the plane is taken at the grazing point. The x-axis is taken along the incident ray passing through G, the light propagation corresponding to increasing xs. The mirror M is defined by an analytical curve $g(x)$:

$$y = g(x) = ax^2 + bx^3 + cx^4 + \cdots \tag{1}$$

We assume that $a > 0$. In this configuration, the mirror and the reflected rays are in the quadrant $x > 0$, $y > 0$.

An incident ray meets the mirror at some point M_1, where it is reflected. Because of the concavity of the mirror, it will meet the mirror farther along, at another point M_2, where it is again reflected. Its successive reflections by the mirror define the points M_1, M_2, M_3, etc. We say that the straight line M_1M_2 is a ray reflected one time. We say that the straight line M_2M_3 is a ray reflected two times, and so on. M_nM_{n+1} is a ray reflected n times. Let us now vary the incident ray. For each value of n, we obtain a congruence of rays reflected n times. This congruence admits a caustic K_n, which is the curve tangent to each ray of the congruence. Our problem is to calculate K_n for all values of the parameter n. It is clear that we have to proceed by recurrence. This means that we have to find successively

- the recurrence relations linking the quantities defined at the step $n+1$ with those defined at the step n;
- the initial values (defined at the step $n=1$);
- the general terms, valid for any n and calculated by using the initial values and the recurrence relations.

3. Recurrence relations

To begin with, we have to define the ray congruences. Let us fix the number n of reflections by the mirror. A ray reflected n times is defined by its endpoint M_n and M_{n+1} on the mirror (see Fig. 2). We may take the coordinate x to parametrize M_n, as well as the direction vector $\mathbf{r}_n$ of the segment M_nM_{n+1}. For $x=0$, one has $M_n=G$. Near the grazing point, x is a small parameter and it will be used in the following for Taylor expansions of the various quantities. Since the starting point $M_n=(x_n, y_n)$ lies on the mirror, we have $x_n(x)=x$ and $y_n(x)=g(x)$. The unit vector $\mathbf{r}_n$ may be defined by its angle ϕ_n with the x-axis: $\mathbf{r}_n(x)=(\cos\phi_n(x), \sin\phi_n(x))$. One has $\phi_n(0)=0$ and the function $\phi_n(x)$ has the form

$$\phi_n(x) = A_nx + B_nx^2 + C_nx^3 + \cdots \tag{2}$$

Introducing a coordinate s along M_nM_{n+1}, we write the congruence of the rays reflected n times as $P_n(x, s) = M_n(x) + s\mathbf{r}_n(x)$.