Optik 158 (2018) 1361–1369

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Statistical characteristics of a Gaussian beam reflected by a retro-reflector in atmospheric turbulence

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ARTICLE INFO

Article history:
Received 19 December 2017
Accepted 10 January 2018

Keywords:
Atmospheric turbulence
Retro-reflector
Degree of coherence
Scintillation index

ABSTRACT

The statistical characteristics of a retro-reflector reflecting a Gaussian beam is analyzed, which shows that the retro-reflector can be approximately characterized by an ABCD ray-transfer matrix, and a monostatic system in atmospheric turbulence with a retro-reflector as the target is processed with ABCD ray matrix theory. Analytical expressions are developed for the basic statistical moments, the degree of coherence and the scintillation index of the retro-reflected wave are calculated in terms of three fundamental statistical moments, and then the backscatter amplification effect associated with these properties are discussed. As comparisons, the results of both monostatic and bistatic cases of a plane mirror are also presented. All the results are based on the weak turbulence with von Karman spectrum.

1. Introduction

A retro-reflector can reflect the incident wave back along the coming path, so it is widely used as a cooperative target to strengthen the intensity of the reflected wave in the monostatic systems, such as the lidar system [1], the laser ranging system and so on. n these applications it is inevitable to involve the problem of light propagation through the turbulent atmosphere in which case the wave front is usually distorted and the performance of the systems [2] may be greatly influenced, therefore, the research of the reflected wave from a retro-reflector in turbulence is necessary.

The atmospheric turbulence may cause the decay of coherence, scintillation of intensity, beam wandering and spreading and so on, which were firstly studied with geometrical optics in the early time in the last century [3,4]; however, the strict condition required for the approach limits its range of application. Later several different theories [5,6], such as Rytov Method, the Markov Equations, Huygens principle and Feynman integral had been developed for light propagation through atmospheric turbulence and all led to analytic results with varying degrees of success, but also were restricted by different difficulties. The most widely used one of these theories is Rytov Method which was introduced by Tatarskii [7] and has been a classical theory until today in condition of weak turbulence [8]. In 1980's, Yura and Hanson revealed that the Huygens–Fresnel integral to study the optical wave propagation through a complex paraxial optical system can be formulated with an ABCD ray matrix [9]. Subsequently, this method was combined with Rytov Method by L. C. Andrews to solve problems in the optical system in atmospheric turbulence [10] and was further extended to solve the double-pass wave propagation when the target is a mirror or a phase screen.

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https://doi.org/10.1016/j.ijleo.2018.01.028
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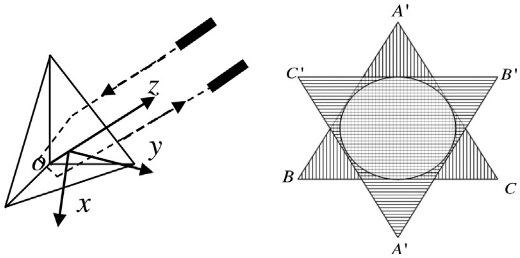

Fig. 1. Optical path of retro-reflector and its effective area.

An interesting phenomenon called the backscatter amplification effect or enhanced backscatter was discovered in 1970's, and later was found widely existing in the double passage problems. In 1980s' with local Green's function Banakh and Mironov had studied extensively this subject and gained results including amplification effect associated with the MCF and the average intensity of the reflected waves from an unbounded plane mirror, a point reflector or a diffuse target for plane wave or spherical wave incidence [11]. L. C. Andrews and his partners also investigated the amplification effect for reflected wave from a mirror or a retro-reflector [12], by dividing the perturbation caused by the turbulence into two parts associated with the folded path and the reciprocal path [13], respectively. But their numerical results were restricted in several special situations.

In this paper we firstly prove that the retro-reflector can be approximately seen as an ABCD optical element, and then the ABCD ray-matrix theory is developed for the double-passage propagation with a retro-reflector as the target. In terms of three basic statistical moments, several second-order and forth-order statistical moments of interest are derived and numerically calculated. Amplification effect associated with the scintillation index is discussed.

2. Property of the retro-reflector

A retro-reflector is a solid tetrahedron composed of a base surface and three right-angle surfaces perpendicular to each other, which can reflect the wave back to the incidence direction. However, not all the wave incident on the retro-reflector can be reflected back. When the wave incidents on the optical surface normally, the effective reflecting area is shown in Fig. 1, the aperture of the reflector is hexagonal [14]. In practice, most retro-reflectors are cut around the inscribed circle. It can be seen that the aperture of the retro-reflector is circular or approximately circular.

In the condition of normal incidence, the performance of the retro-reflector reflecting a wave is rotational symmetrical [15], so it can be characterized by an ABCD ray-transfer matrix. If a ray incidents on the retro-reflector in position x_1 in direction x_1', the position and the direction of the output ray are

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \tag{1}$$

The ray matrix of the retro-reflector without considering the edge effect is

$$\begin{pmatrix} A_{\text{inf}} & B_{\text{inf}} \\ C_{\text{inf}} & D_{\text{inf}} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2}$$

Assuming that the retro-reflector has a Gaussian aperture with effective aperture radius W_R, and by introducing the notation $\alpha_R = 2/\left(kW_R^2\right)$, a finite retro-reflector can be modeled as a connection of a Gaussian lens and an infinite retro-reflector.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ i\alpha_R & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -i\alpha_R & -1 \end{pmatrix} \tag{3}$$

3. Wave propagation by ABCD ray matrix

In a monostatic system shown in Fig. 1, wave propagates forward and backward through the same path, for convenience, the coincident paths are unfolded into the same direction as shown in Fig. 2. The optical field of a Gaussian-beam wave of unit amplitude at the emitting aperture of a transmitter can be expressed by [6]

$$U_0(\boldsymbol{r}, 0) = \exp\left(-\frac{1}{2}\alpha k r^2\right) \tag{4}$$

where $\alpha = 2/kW_0^2 + i/F_0$, k is the optical wave number, W_0 is the effective beam radius, F_0 is the phase front radius of curvature, $\boldsymbol{r}$ is a vector transverse to the propagation axis. The Gaussian–beam turns to a plane wave if $\alpha = 0$, or a spherical wave if $\alpha = \infty$.