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Modeling of elastic finite-length space rolling-sliding contact problem

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ABSTRACT

This paper presents a semi-analytical method (SAM) to solve sliding finite-length space contact problem with consideration of the effect of free end surface. The equation set based on the overlapping concept is analytically solved with tangential load considered and the explicit expressions for equivalent mirrored loads are obtained. Conjugate gradient method (CGM) is adopted to solve the pressure distribution and Fast Fourier transform (FFT) is used to speed up stress related calculation. The model is verified by FEM and shows a high conformity. Different line contact situations are discussed to explore the effect of free end surfaces.

1. Introduction

It is quite common in industrial applications that there are two free end surfaces perpendicular to the axis of part, especially in some mechanical elements such as gears, cams and roller bearings, and in these contact situations, sliding motion usually exists that makes working condition more severe. The dimension of both the contacting parts along the axis are actually finite, however, the widely-used Hertz theory of contact is based on half space, through which only the approximate solution can be obtained, especially in the vicinity of the edge. For accurate application, it is desirable to consider the effect of the two free boundaries, i.e., finite-length space contact problem.

Finite-length space contact problem is the extension of quarter-space contact problem, and many researchers have developed their study on the latter in the past decades. Among them an enlightening idea is proposed by Hetenyi [1], who employed an iterative scheme to calculate the stresses in an elastic quarter space subjected to a concentrated normal load on its surface. Two half-spaces with mirrored loads are overlapped to free the expected boundary from normal stress. Then some researchers extended the investigations with overlapping method. Gerber [2] studied the contact problem of quarter-space provided that the indenter was not located too close to the edge. Keer et al. published a series of researches on quarter space [3–7]. They firstly analyzed point normal and tangential loading of a quarter-space using the overlapped-half-space idea of Hetenyi, then extended their analysis to a frictionless contact problem and elastic-plastic rolling-sliding contact problem. It is illustrated that the friction coefficient contributes less to the change of the residual lateral displacement than the edge effect. Recently, Wang et al. studied the two joined quarter spaces pressed by a rigid sphere in the view of equivalent inclusion method [8] and showed that the von Mises stress increases in quarter space. Besides their works, different method were also proposed to solve this problem, such as Ritz's method by Guenfoud et al. [9], the fast correction method by Guilbault et al. [10,11] and a simple adjustment method by Kalker et al. [12]. Recently Zhang et al. [13] proposed an explicit solution for the elastic quarter-space problem with normal loading based on overlapping method and the matrix formulation, which improved the computational efficiency. Following Zhang's work, the present authors [14] proposed a new overlapping model to study the dry contact between a roller and a finite-length space, which considers the effect of two free end surfaces. The main previous works related with free end surface are summarized in Table 1 for clarity.

Sliding contact is quite common, sometimes inevitable, in practical applications. Based on fundamental solutions for interior stress by normal and tangential pressure on a rectangular patch [15], many researchers investigated various contact situations, such as rough surface contact [16], partial slip [17], coatings [18] and EHL analysis [19]; however, mainly based on half space assumption. As shown in Table 1, few studies have been done on sliding contact problem with consideration of free boundary, and much less for finite-length space.

Through semi-analytical method (SAM), the present paper improves the overlapping method used in [14] to consider the tangential load and focuses on the normal and sliding contact of finite-length space to show the different features of pressure and stresses distribution compared with half-space. The explicit solution of equivalent loads on half spaces is firstly derived based on formulation method; then the normal and sliding contact problem between rigid roller with various profiles and finite-length space is solved by conjugate gradient method (CGM) and the discrete convolution-Fast Fourier transform (DC-FFT);

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Nomenclature

a	Hertzian contact half-width
e	distance from contact border to free end surface
$\mathbf{E}$	unit matrix
E^*	equivalent Young's modulus
f	friction coefficient
G	equivalent shear modulus
L_f, L_c	length of finite-length space and roller
M, N, L	the number of grids along x, y, z directions
MA, **MB**, **MC**	coefficient matrix from load P(x, y) to $\mathbf{P_{h1}}$, $\mathbf{P_{v1}}$, and $\mathbf{P_{v2}}$
$\mathbf{M_{i-j}}$	influence matrix from load on surface i to stress on surface j
p_0	maximum Hertzian pressure
$P(x, y)$	applied pressure distribution
$\mathbf{P_{hi}}$, $\mathbf{P_{vj}}$	array of equivalent load on half-spaces, i=1, 2, 3, j=1, 2
r_0	radius of roller fillet
Ra	radius of roller on mid-length
$\mathbf{S}$	array of stress components
$\boldsymbol{s_l}$	the height of generatrix of Lundberg profiled roller
$T(x,y)$	friction load distribution, T=$f \cdot P$
$\boldsymbol{u}$	surface elastic deformation
W	force applied on the roller
β	half cone angle of tapered roller
γ	length-radius ratio of finite-length space model, γ=L_f/Ra
δ	modification factor for Lundberg profiled roller
μ	Poisson's ratio
$\boldsymbol{\sigma_{ij}}$, $\boldsymbol{\tau_{ij}}$	stresses component
ω	revolution angular velocity of roller
I, II	end free surface of finite-length space on left and right side

Table 1
List of some of the work involving free boundary.

Author	Quarter or finite-length space	Shear traction	Contact problem	Method
Keer et al. [6,7]	Quarter	Yes	Yes	Overlapping and iterative method
Wang et al. [8]	Quarter	Yes	Yes	Equivalent inclusion method
Zhang et al. [13]	Quarter	No	No	Overlapping and formulation method
Guenfoud et al. [9]	Quarter	No	No	Ritz's method
Guilbault et al. [10,11]	Finite-length	No	Yes	Overlapping and Correction method
Zhang et al. [14]	Finite-length	No	Yes	Overlapping and formulation method

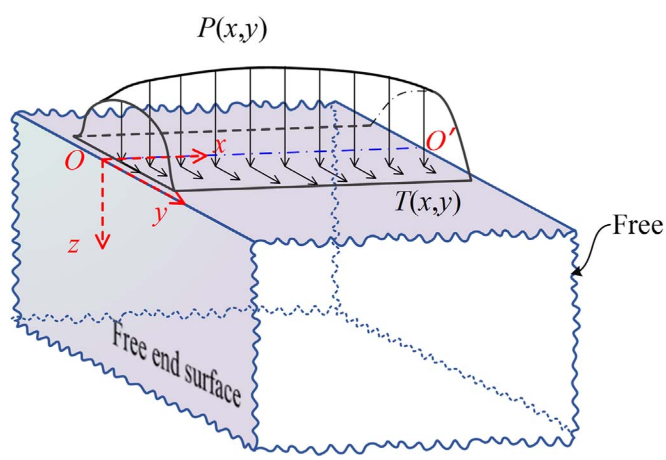

Fig. 1. Schematic plot for finite-length space problem.

the results show that this method is capable to solve the finite-length space problem accurately and efficiently compared with FEM, then, the effect of free surfaces is investigated.

2. Theoretical model

2.1. Overlapping model

The overlapping method described in [14] is used for normal contact of finite-length space. In order to consider the tangential load, a new overlapping process should be proposed. Similarly, a finite-length space with top surface normally and tangentially loaded and two end surfaces free is treated as the kernel of the new finite-length space problem, as shown in Fig. 1. Here $P(x, y)$ is an arbitrary pressure applied on the top surface; $T(x, y)$ is tangential load along y-axis (sliding direction) equal to the production between contact pressure $P(x, y)$ and friction coefficient f, while the friction force along x-axis (axial direction of roller) is neglected.

Keer et al. investigated the rolling-sliding contact of quarter-space and presented the overlapping process in which tangential load is involved [7]. Following Keer's ideal of overlapping method for quarter-space, in order to remove the stresses on the two free end surfaces of finite-length space, the overlapping scheme is extended as shown in Fig. 2. A horizontal (H) half space is divided into three parts A, B and C by the two expected free end surfaces (named I and II), and the vertical (V) half space with free surface I as shown in Fig. 2(b) is divided into two parts A and D by the horizontal middle surface (named 1); and so

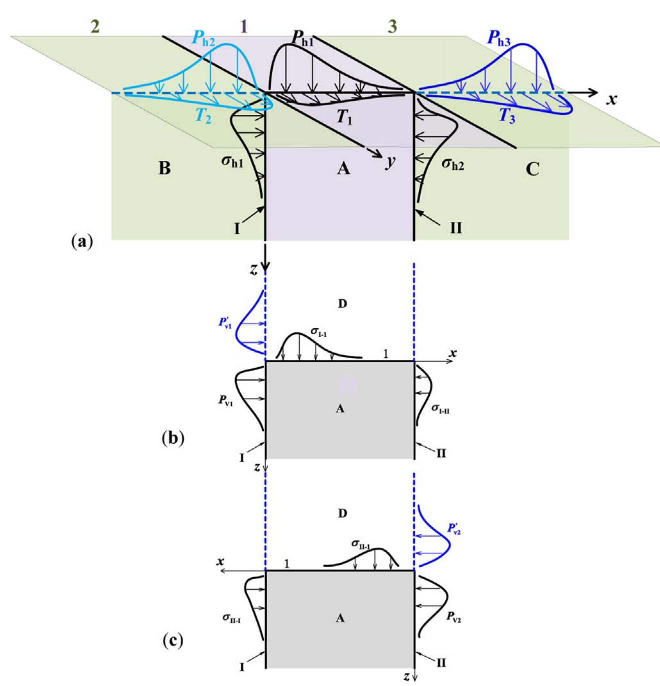

Fig. 2. The loading condition of the three half-spaces for overlapping: (a) the three parts of horizontal half-space and (b) the vertical half-space for free surface I and (c) the vertical half-space for free surface II.