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Discrete Optimization

Exact solution of hub network design problems with profits

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ABSTRACT

This paper studies hub network design problems with profits. They consider a profit-oriented objective that measure the tradeoff between the revenue due to served commodities and the overall network design and transportation costs. An exact algorithmic framework is proposed for two variants of this class of problems, where a sophisticated Lagrangian function that exploits the structure of the problems is used to efficiently obtain bounds at the nodes of an enumeration tree. In addition, reduction tests and partial enumerations are used to considerably reduce the size of the problems and thus help decrease the computational effort. Numerical results on a set of benchmark instances with up to 100 nodes confirm the efficiency of the proposed algorithmic framework. The proposed methodology can be used as a tool to solve more complex variants of this class of problems as well as other discrete location and network design problems involving servicing decisions.

1. Introduction

Large-scale transportation and telecommunications networks arising in air and ground transportation, postal delivery, and rapid transit systems frequently use hub-and-spoke architectures to efficiently route flows. Transshipment, consolidation, or sorting points, referred to as hub facilities, are employed in these networks to connect a large number of origin/destination (O/D) pairs indirectly by using a small number of links. Broadly speaking, *hub location problems* (HLPs) consider the design of hub networks by selecting a set of nodes to locate hubs, assigning a set of O/D nodes to hubs, and routing a predetermined set of commodities through the network while optimizing a cost-based (or service-based) objective function. An important feature of HLPs is that some design decisions (i.e. link activation and routing) are implicitly determined by the assumptions made on the cost structure and the assignment pattern. *Hub arc location problems* (HALPs) extend HLPs to explicitly consider non-trivial design decisions on the activation of hub arcs to efficiently interconnect hub nodes.

This paper studies *hub network design problems with profits* (HNDPPs), a class of HALPs recently introduced in Alibeyg, Contreras, and Fernández (2016). HNDPPs relax the classical requirement of most HLPs that all service demand must be satisfied, and incorporate one additional level to the decision making process so as to determine the O/D nodes and associated commodities whose demand must be served. The rationale behind HNDPPs is that in many applications a revenue is obtained for serving the demand of a given commodity. Capturing such a revenue is likely to incur not only a routing cost but also additional setup costs, as the O/D nodes of the served commodities may require the installation of additional infrastructure. Classical HLPs, however, ignore these considerations, as reflected by the requirement that the demand of every commodity must be served. Broadly speaking, this requirement expresses the implicit hypothesis that the overall cost of solution networks will be compensated by the overall revenue (or by a requirement to serve all OD pairs, as for postal systems). Since such hypothesis does not necessarily hold, incorporating decisions on the nodes where service should be offered and the commodities that should be routed have important implications in the strategic and operational costs.

Potential transportation applications of HNDPPs arise in the airline and ground transportation industries. In the case of airline companies, network planners have to design their transportation network when they are first entering into the market, or may have to modify already established hub-and-spoke networks through alliances, mergers and acquisitions of companies. The decisions are to determine: (i) the cities (nodes) that will be served, (ii) the O/D pairs that will be served, (iii) the location of the hub airports, (iv) the flight legs to be used to connect regional airports (served nodes) to hub airports, and (v) the flight legs to connect hub airports to each other, and (vi) the transportation route for each O/D

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pair.The objective is to find an optimal hub network structure that maximizes the total net profit for providing air travel services to a set O/D pairs while taking into account the (re)design cost of the network.

Alibeyg et al. (2016) introduced and analyzed several variants of HNLPPs. These models incorporate into the decision-making process additional strategic decisions on the nodes and the commodities that will be served. They consider profit-oriented objectives that measure the tradeoff between the revenue due to served commodities and the overall network design and transportation costs. Broadly speaking the proposed HNDPPs are of three types: (i) primary profit-oriented models, which may or may not require to provide service to all O/D pairs for which both the origin and destination are served (i.e. service commitment constraints), (ii) profit-oriented models with network design decisions that incur setup costs on the edges used on service routes, and (iii) more complex models with multiple demand levels and possibly multiple service levels as well. The results of extensive computational experiments reported in Alibeyg et al. (2016) illustrate the characteristics of the solution networks produced by these different models, as well as the computational difficulty for solving them with a state-of-the art commercial solver. In particular, the results also show that the proposed formulations are very demanding from a computational point of view, in terms of both computing time and memory when used with a commercial solver. For the primary profit oriented model without any additional service commitment constraints, instances with up to 70 nodes can be solved to optimality in one day of CPU time, and when such additional constraints are added only instances with up to 60 nodes can be solved. When approaching the more complex models with multiple demand and service levels, only instances with up to 35 nodes can be solved in the same time limit.

In this paper we focus on methodological aspects leading to the exact solution of the two primary HNDPPs presented in Alibeyg et al. (2016). The first one, denoted as PO_1, is flexible in the sense that among all commodities associated with served O/D nodes, only those that are actually profitable are routed. It is applicable in situations where there are no service commitments or external regulations forcing the decision maker to serve any commodity whose O/D nodes are both activated. The second model, denoted as PO_2, considers a more restrictive scenario in which such commitments or regulations do exist and thus, all commodities whose O/D nodes are both activated would have to be served, even if this would reduce the total profit.

The main contribution of this paper is to propose a unified algorithmic framework applicable to large-scale instances of both PO_1 and PO_2 models involving up to 100 nodes. It is an exact branch-and-bound procedure in which a sophisticated Lagrangian relaxation is used to obtain tight bounds at each node of the enumeration tree. In particular, the proposed Lagrangian function resorts to the solution of well-known *quadratic boolean problems* (QBPs). We show how, due to the special cost structure associated with the quadratic term of the objective function, the QBPs can be efficiently solved by transforming them to classical *minimum cut problems*. The algorithm is enhanced through several algorithmic refinements that make it more efficient. These include: (i) variable elimination techniques that allow reducing considerably the size of the formulations at the root node, (ii) a partial enumeration phase capable of effectively exploring the solution space by reducing the required number of nodes in the tree, and (iii) the use of simple but effective primal heuristics embedded in the subgradient algorithm that exploit the structure of the problem. Computational experiments confirm the effectiveness of our exact algorithmic framework since it is able to obtain optimal solutions for instances with up to 100 nodes for both PO_1 and PO_2, whereas a commercial solver can only handle instances with up to 70 and 60 nodes, respectively.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature related to HNDPPs. In Section 3 we introduce the formal definition and mixed integer programming (MIP) formulations of PO_1 and PO_2. Section 4 describes the proposed Lagrangian relaxations of PO_1 and PO_2 and the solution of their associated Lagrangian duals. Section 5 explains the variable elimination techniques used whereas Section 6 presents the partial enumeration and the overall branch-and-bound algorithm. Section 7 describes the computational experiments we have run. Conclusions follow in Section 8.

2. Literature review

HNDPPs extend *hub arc location problems* (HALPs) by selecting the nodes to be served and the commodities to be routed. That is, HNDPPs incorporate an additional level to usual HALP decisions. In its turn, HALPs extend fundamental HLPs (see, Campbell & O'Kelly, 2012; Contreras, 2015) by incorporating network design decisions dealing with the selection of the hub arcs that can be used in O/D paths, in addition to classical hub location and allocation decisions. Different HALPs have been studied in the literature. For instance, HALPs with a cardinality constraint on the number of opened hub arcs (Campbell, Ernst, & Krishnamoorthy, 2005), HALPs that incorporate setup costs for the hub nodes and hubs arcs (Contreras & Fernández, 2014; Gelareh, Monemi, & Nickel, 2015), or HALPs that impose particular topological structures, such as tree–star (Contreras, Fernández, & Marín, 2010), star–star (Labbé & Yaman, 2008), ring–star (Contreras, Tanash, & Vidyarthi, 2016), and hub lines (Martins de Sá, Contreras, & Cordeau, 2015; Martins de Sá, Contreras, Cordeau, Saraiva de Camargo, & de Miranda, 2015). We can also relate HNDPPs to studies that focus on the design of hub networks in airline transportation (see, for instance, Aykin, 1995; Jaillet, Song, & Yu, 1996; Sasaki, Suzuki, & Drezner, 1999; Bryan & O'Kelly, 1999; O'Kelly, 2012; Saberi & Mahmassani, 2013). We note that all these works focus on the location of hubs, but ignore other relevant decisions addressed in HNDPPs, like the nodes to be served and the commodities to be routed.

Contrary to most HLPs and HALPs that optimize a cost-based (or service-based) objective, HNDPPs optimize a profit expressed as the difference between the revenue obtained for the service offered and the costs due to the design of the network and to transportation. This feature relates HNDPPs to two families of HLPs, aiming at the maximization of the profit obtained for serving nodes and routing commodities: *maximal hub covering problems* (MHCPs), and *competitive hub location problems* (CHLPs). MHCPs is the most relevant class of HLPs to HNDDPs among the existing coverage models and assume demand is covered if both origin and destination nodes are within a specified distance of a hub node. These problems were introduced by Campbell (1994) and more recently extended by Hwang and Lee (2012) and Lowe and Sim (2012). Similarly to HNDPPs, MHCPs allow some commodities not to be served (in this case due to covering constraints). However, like in the previous HLPs mentioned above, MHCPs do not incorporate decisions on the nodes to be served, which are essential in HNDPPs.

From a different perspective, CHLPs focus on the design of hub networks within the framework of competing firms. Most CHLPs assume that a company already operates in the market (leader), and address the maximization of demand captured by a new company who wants to enter the market (follower). That is, the usual objective in CHLPs is to maximize the market share of the new firm. Marianov, Serra, and ReVelle (1999) introduce CHLPs with two competitors in which the follower looks for the best location