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Analytical pricing of discretely monitored Asian-style options: Theory and application to commodity markets

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Abstract

We compute an analytical expression for the moment generating function of the joint random vector consisting of a spot price and its discretely monitored average for a large class of square-root price dynamics. This result, combined with the Fourier transform pricing method proposed by Carr and Madan [Carr, P., Madan D., 1999. Option valuation using the fast Fourier transform. Journal of Computational Finance 2(4), Summer, 61–73] allows us to derive a closed-form formula for the fair value of discretely monitored Asian-style options. Our analysis encompasses the case of commodity price dynamics displaying mean reversion and jointly fitting a quoted futures curve and the seasonal structure of spot price volatility. Four tests are conducted to assess the relative performance of the pricing procedure stemming from our formulae. Empirical results based on natural gas data from NYMEX and corn data from CBOT show a remarkable improvement over the main alternative techniques developed for pricing Asian-style options within the market standard framework of geometric Brownian motion.

JEL classification: C63; G13

Keywords: Asian options; Discrete monitoring; Laplace transform; Fourier transform; Commodity markets; Energy markets

1. Introduction

Asian-style options, and other options written on alternative definitions of average prices, are effective hedging devices in commodity markets. As reported by Eydeland and Wolyniec (2003), these derivatives play an important role in price risk management performed by local delivery companies in the gas market. Moreover, oil markets often use these securities to stabilize cash flows that stem from meeting obligations to clients.

The market model for pricing Asian-style options is the geometric Brownian motion. This process is fraught with two major shortcomings. First, the assumption of normal price returns does not reflect the empirical features displayed by the vast majority of time series of commodity quotes. As illustrated by Richter and Sorensen (2000), Eydeland and Wolyniec (2003), and Regnier (2007), among others, these latter exhibit variable degrees of time varying and price dependent volatility functions. Second, there is no closed-form expression for the arbitrage-free price of options on arithmetic averages of lognormally distributed prices. Time consuming price approximations must be implemented, sacrificing both precision and time in the resulting procedure (Wilmott et al., 1993). In this respect, Fusai and Roncoroni (2008) provide a detailed comparison of alternative pricing methods for Asian-style options written on price dynamics following a geometric Brownian motion.

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We start by assuming that asset price dynamics are driven by a square-root process in the spirit of Cox et al. (1985). This process subsumes important elements characteristic of commodity price series while preserving analytical tractability. Moreover, it can easily be extended such that the resulting evolution fits the market forward curve, the time pattern of spot price volatility and mean reversion.

On the theoretical side, we derive an analytical formula for pricing discretely monitored Asian-style options in the above mentioned setting. To achieve this goal, we follow a two step procedure: first, we compute the moment generating function of the joint pair consisting of the commodity spot price $S_{n\Delta}$ at a future maturity $n\Delta$ and the weighed cumulated price $\sum_{j=0}^{n}\alpha_j S_{j\Delta}$ over the discretely monitored time horizon $\{0, \Delta, \ldots, n\Delta\}$; then, we apply a computational pricing approach based on the Fourier transform, as proposed by Carr and Madan (1999). The ability to compute an analytical expression for the underlying pair $(S_{n\Delta}, \sum_{j=0}^{n}\alpha_j S_{j\Delta})$ constitutes the main theoretical result obtained in this paper in relation to the existing literature in the field. Incidentally, we note that discrete monitoring definitively represents a more realistic assumption than continuous monitoring. Moreover, it allows us to compute the transform of the joint distribution of the absolute and cumulated spot prices using a simple recursive procedure. A striking result is that the mentioned transform can be obtained using only the transform of the underlying commodity price.

We also present three important extensions of our main formula: first, we let the underlying spot price process exhibit a time dependent drift, a property allowing the resulting dynamics to recover the quoted set of forward prices; second, we adopt a time varying volatility coefficient, a feature allowing our model to fit either the term structure of implied volatilities or a time dependent, *e.g.*, seasonal and spot price historical volatility; third, we consider spot price dynamics exhibiting mean reversion in their trend, a quality shown by some important classes of commodity prices, among which we cite agriculturals and energy-related products such as electricity and gas. These extensions represent a further theoretical innovation on the pricing of Asian-style options compared to the published literature (Dassios and Nagaradjasarma, 2006).

On the empirical side, we perform four experiments aimed at assessing the absolute and relative quality of our pricing device. First, we measure the extent to which prices computed using discrete monitoring deviate from figures resulting from those obtained using formulae for the continuous monitoring case. We see that convergence is approximately linear in the number of monitoring dates. The resulting rate of convergence underpins the use of a fast, accurate method of pricing discretely monitored Asian-style options such as the one we propose herein.

Next, we compare prices obtained using the standard Black–Scholes model to the ones stemming from implementing our formulae. The former are obtained using two methods proposed in the literature on the subject. The latter are computed using a volatility assessment for our square-root model that is consistent with the volatility parameter in the geometric Brownian motion used to feed in the alternative methods mentioned above. This procedure makes our price directly comparable to the others. Our pricing device proves to be rather quick to obtain, whereas alternative pricing algorithms are always much slower to perform. Moreover, it mostly offers results which lie within the alternative methods used to approximate the option price under the market model.

Then, we measure the impact of including market information about the forward prices into the spot price dynamics for the purpose of pricing Asian-style options. We perform this analysis using quotes taken from the Natural Gas Market at NYMEX. It turns out that a non-flat forward curve produces highly significant option price deviations from figures obtained in the case where such information is not accounted for by the underlying spot price model.

Finally, we assess the impact of including information about the time structure of historical volatility into our pricing device. We perform a test on corn price data quoted at CBOT. It turns out that using this information may result in significant price discrepancies compared to the quotes obtained using the market model represented by the geometric Brownian motion. Our results suggest that when pricing Asian-style options in market contexts where a seasonal component strongly affects the evolution of spot price volatility, one should include this information as precisely as possible. This remark is particularly important for several commodity markets, such as energy and agriculturals, where the time variation of volatility is significantly pronounced.

The paper is organized as follows. Section 2 derives a closed-form expression for the moment generating functions of the underlying commodity price $S_{n\Delta}$ and the pair $(S_{n\Delta}, \sum_{j=0}^{n}\alpha_j S_{j\Delta})$. Section 3 extends these results to the case of spot price dynamics that fit a quoted forward price curve, a time varying volatility structure, and a mean reverting behavior. Section 4 relates these expressions to the Laplace transform of the fixed strike Asian-style option price and the Fourier transform of the floating strike Asian-style option price. Section 5 performs numerical experiments on gas data taken from NYMEX. Section 6 concludes with a few comments and suggestions for future development.

2. Recursive valuation of the underlying pair transform

We consider spot price dynamics driven by a simple square-root process under the (possibly selected) risk-neutral probability measure[1]:

[1] If the underlying market is incomplete, as is the case for non-storable or partly storable commodities such as electricity or perishable goods, a variety of equivalent probability measures are compatible with the assumption of absence of arbitrage opportunities. In this case, the user has to adopt an appropriate method for selecting one of these pricing measures for pricing purposes. In our model setting, we assume that a measure is selected within this class at the outset.