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Journal of Economic Theory 173 (2018) 18–55

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

On the existence and uniqueness of stationary equilibrium in Bewley economies with production ☆

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Received 2 May 2016; final version received 14 September 2017; accepted 16 October 2017
Available online 23 October 2017

Abstract

I prove existence of stationary recursive competitive equilibrium in Bewley economies with production under specifications in which (i) the utility function is allowed to be unbounded, and (ii) the underlying discrete idiosyncratic productivity process can take any form, aside from mild restrictions. Some of the intermediate results provide theoretical basis for assumptions often made in the quantitative macroeconomics literature. By providing an example, I illustrate that the equilibrium is not necessarily unique, even under typical specifications of the model, and discuss the underlying reasons for multiplicity.

JEL classification: E21; C61; C62; D31

Keywords: Bewley–Aiyagari model; Recursive competitive equilibrium

1. Introduction

A large body of macroeconomics literature is devoted to the study of income and wealth inequality and their impact on macroeconomic variables. While economic models without fric-

☆ I am grateful to Jess Benhabib, Gaetano Bloise, Alex Citanna, William Hawkins, Jim Kahn, Baris Kaymak, Dirk Krueger, John Stachurski, and two anonymous referees for their detailed comments on an earlier version of this paper. I also would like to thank seminar participants at Cowles Foundation 12th Annual Conference on General Equilibrium and Its Applications for their feedback. An earlier version of this paper circulated under the title "On the Existence of Equilibrium in Bewley Economies with Production".

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https://doi.org/10.1016/j.jet.2017.10.006
0022-0531/

tions proved useful in answering many macroeconomic questions of interest, due to their stark prediction of "no cross-sectional heterogeneity", they fell short of analyzing any sort of inequality.[1] Stepping outside the boundaries of representative-agent/complete-markets paradigm, many economists working on inequality used Bewley (1984, 1986) models to capture the interplay between financial frictions, cross-sectional heterogeneity, and macroeconomic variables. This class of models has been used extensively to analyze excess sensitivity of consumption to temporary changes in income, equity-premium and "low risk-free rate" puzzles, the relationship between micro and macro estimates of labor supply elasticity, the welfare costs of inflation and business cycles, as well as normative and positive implications of income taxation.

Despite this vast literature that dates back to the 1970s, the issue of existence of equilibrium in these models remains unresolved except under restrictive assumptions that do not correspond to those commonly made in actual applications. To the best of my knowledge, a complete proof of existence of stationary recursive competitive equilibrium in the canonical Bewley model with production, i.e. Aiyagari (1994) model, is still missing from the literature. Some proofs are available in slightly different environments and/or under restrictive assumptions such as bounded utility, and i.i.d. shocks. However, even for a "textbook version" of the model that features constant-relative-risk-aversion (CRRA) utility function, or productivity shocks that exhibit some persistence, the existence of an equilibrium is simply *assumed*, but not rigorously established.

Motivated by this shortcoming, in this paper I provide a proof of existence of stationary recursive competitive equilibrium for Aiyagari (1994) model. From a practical point of view, the proof covers (i) any type of discrete idiosyncratic productivity process, provided that it satisfies the mild restriction that the lowest idiosyncratic productivity state exhibits some persistence, (ii) the cases in which the utility function is unbounded, either from above, and/or from below.

Many technical challenges are addressed. Building on several theoretical results in a recent paper by Li and Stachurski (2014), I use Euler equation-based methods, motivated by Coleman (1990) "policy iteration" approach to establish the existence and uniqueness of the solution to the households' problem. This approach renders characterization of policy functions, the most important objects of analysis, more direct and convenient. This is particularly relevant for the main existence result in this paper, as some of the key steps of the proof actually rely on the theoretical properties of the policy function mapping (referred to as "Coleman operator" throughout). Second, this approach has the advantage that one can focus on the properties of the marginal utility function, effectively eliminating the need to impose strong restrictions on the *level* of the utility function, e.g. boundedness, an arguably restrictive feature violated by most functions used in the applied literature.

The first step I take to establish the existence and uniqueness of the stationary distribution is to generalize a well-known early result of boundedness of the state space under i.i.d. shocks, by Schechtman and Escudero (1977). I show that the same property holds under *weaker* assumptions on preferences, and for arbitrary Markov processes. Second, since the Markov process is allowed to be non-monotone, the proof of existence and uniqueness of stationary distribution does not (and cannot) rely on monotonicity of the policy functions with respect to the idiosyncratic productivity. The key result used in the several steps of the proof is the fact that every household with finite wealth level is eventually borrowing constrained with positive probability. I show that every agent runs down assets in the lowest productivity state, independent of whether the household is impatient with respect to the interest rate or not. This follows as a simple conse-

[1] Unless, of course, ex ante heterogeneity is exogenously imposed, for instance, by using heterogeneous preferences.