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Patterns of anxiety in algebraic problem solving: A three-step latent variable analysis

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ABSTRACT

Math anxiety is associated with avoidance of math in high school. It is commonly assessed using questionnaires that examine anxiety experienced for arithmetic. A more nuanced understanding of math anxiety would be gained by investigating the anxiety associated with math problems encountered in high school at the time they are encountered. To this end, we investigated the anxiety associated with algebraic problem solving in 14-year-olds. We varied problem difficulty and the time allowed to solve problems, and assessed students' anxiety concurrently as they solved problems. In two studies, latent variable mixture modelling revealed meaningfully different patterns of algebra ability and anxiety relationships that changed as a function of problem difficulty and time pressure. The results highlight the value of using latent variable mixture models to identify subgroup patterns of abilities and caution against making overly general claims about the role of anxiety in math problem solving.

1. Introduction

Math anxiety affects math problem solving abilities and avoidance of math in high school and beyond (Ashcraft, 2002; Dowker, Sarkar, & Looi, 2016). Claims for the impact of math anxiety on problem solving are largely based on self-reports of anxiety experienced solving arithmetic problems (Ashcraft & Kirk, 2001; Dowker et al., 2016; Lyons & Beilock, 2012; Richardson & Suinn, 1972). These claims may be limited in two respects. First, high school students report higher stress for more complex math (e.g., algebra), compared to arithmetic problems (Chinn, 2009). Second, large variations in high school students' anxiety and math problem solving abilities caution against making general statements about math-anxiety associations (Trezise & Reeve, 2017). Arguably, a more nuanced understanding of math anxiety in high school students and its sequelae would be gained by investigating the anxiety experienced solving math problems encountered in high school. Algebra, for example, is known to be troublesome for many high school students (Kieran, 1992). In the present study we employed latent variable mixture modelling to classify students' algebraic problem solving ability—anxiety level relationships, the aim of which was to better understand variations in the nature of math anxiety in high school students.

1.1. Math anxiety

In research on general anxiety, anxiety is viewed as a trait as well as a reaction to situations/contexts that affect cognitive functioning (Epstein, 1971; Gross, 1998; Schmeichel, Volokhov, & Demaree, 2008; Spielberger, 1985). Math anxiety, in contrast, is most often viewed as a trait, and the situational factors that elicit it are examined less often. It would seem important to specify the latter factors (e.g., problem complexity, examination pressure) to more fully understand the significance of trait and state math anxiety.

Math anxiety is affected by problem solving context. Beilock and Carr (2005), and Beilock and DeCaro (2007), for example, show math anxiety is affected by task pressure. In their work, Beilock and DeCaro (2007) examined the relationship between arithmetic problem solving, task pressure and working memory. Pressure was manipulated by varying financial rewards and social incentives to represent high and low anxiety. Findings showed under low pressure, students used high-cognitive demand strategies (working memory-demanding rule-based algorithm) to solve the problems. In contrast, under high pressure conditions, they used low-cognitive demand strategies. This finding suggests pressure affects math anxiety that, in turn, affects math problem solving.

Math anxiety changes with age (Dowker et al., 2016). Several studies have shown math anxiety increases from childhood into adolescence (Dowker et al., 2016; Hill et al., 2016; Wigfield & Meece, 1988).

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Wigfield and Meece (1988) and Hembree (1990), for example, found worry about math is highest at Grade 9. Several reasons have been suggested for this age-related change, including increases in general anxiety, increased exposure to negative math attitudes and stereotype threat, and changes in working memory (Dowker et al., 2016). It is also likely math anxiety varies in reaction to math problem complexity.

1.2. Stability of math anxiety

Math anxiety changes over short time periods. Trezise and Reeve (2016) examined the degree to which 14-year-olds' math anxiety and cognitive abilities interact with each other over time, and the significance of these changes for algebraic problem solving ability. Trezise and Reeve (2016) assessed students' algebraic anxiety and working memory[1] several times over the course of a single day. A latent change score analysis examined differences in intra-individual changes in working memory and anxiety. Findings showed high anxiety predicted decreases in working memory and low working memory predicted increases in anxiety. The findings also showed students with high working memory and low initial anxiety (assessed as worry) were likely to show low anxiety over time. Students with higher initial worry and/or low working memory were likely to show increases in worry over time. These findings suggest math anxiety fluctuates as a function of context. Indeed, in an earlier study Trezise and Reeve (2014a) assessed patterns of stability/change in math anxiety-cognition relationships. Specifically, they assessed changes in 14-year-olds' anxiety and working memory over a single day as the students prepared for an algebraic problem solving test. On the basis of findings from a latent transition analysis, they found at first assessment most students belonged to a high working memory (with low or high anxiety) profile, or a moderate working memory-low anxiety profile. Students in the high working memory-low anxiety profile were likely to remain in that profile over time. Students in the high working memory-high anxiety profile and moderate working memory-low anxiety profile were likely either to remain in the same profile or move to a profile with lower working memory and/or higher anxiety. The different working memory-anxiety profiles differentially predicted algebraic problem solving abilities.

Three points are worth noting about the Trezise and Reeve (2014a, 2016) research. First, latent variable modelling identified the reciprocal and changing relationships between math anxiety, working memory and math problem solving. More conventional analytical methods that assume linear relationships among factors would not have revealed these relationships (see Bergman, Magnusson, & El-Khouri, 2003; MacCallum, Zhang, Preacher, & Rucker, 2002 for similar claims).

Second, the findings suggest math anxiety and working memory differ among individuals and may change in different ways over time. Specifically, different math anxiety and working memory interacted in meaningful ways over time. While these findings provide insight into the complexity of changing math anxiety-cognition relationships, the authors did not examine how problem difficulty and/or pressure affect math anxiety and, in turn, algebraic problems solving ability—which is the focus of the present study.

Third, the research shows students with poor math abilities are likely to exhibit math anxiety, which in turn impairs their working memory and which in turn, affects their math problem solving ability. These findings are consistent with Eysenck and colleague's research, which showed general anxiety affects cognition by reducing available working memory capacity (Eysenck & Calvo, 1992; Eysenck, Derakshan, Santos, & Calvo, 2007), as well as research that reports a relationship between working memory and math ability (see Raghubar, Barnes, & Hecht, 2010).

The interaction between working memory and anxiety, however, is likely to be affected by other factors. Ashcraft and Faust (1994), for example, found mental arithmetic problem solving speed and accuracy differed as a function of anxiety level. A moderately high math anxiety group showed slow problem solving speeds and moderate accuracy. A high anxiety group sacrificed accuracy for speed solving complex, but not simple problems. Similarly, Trezise and Reeve (2014a) showed highly math anxious students with poor working memory capacity were fast and inaccurate algebraic problem solvers, compared to students with good working memory. These findings suggest the impact of math anxiety on problem solving may reflect an interaction between working memory, and math task demands.

1.3. Algebra problem solving

Algebra competence is important for advanced math. Kieran (1992) argues difficulties learning to solve algebraic equations stem from a lack of understanding of the structure of algebra. Equivalence and negative number concepts are fundamental to algebraic reasoning. Mathematical equivalence reflects the principle that values of the left and right side of the equal sign are equivalent (Kieran, 1981). The negative sign may indicate an operation of subtraction or signify a number has a negative value (Gallardo & Rojano, 1994; Vlassis, 2004). Simple concepts of the equal and negative signs are introduced early in primary/elementary school years and more sophisticated concepts introduced later in primary years. An understanding of equivalence occurs in several stages: (1) operations on the left of the equal sign, $a + b = c$ (*Canonical* equations); (2) operations on the right, $c = a + b$ (*Non-Canonical* equations); and (3) operations on both sides, $a + b = c + d$ (*Relational* equations) (Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). Subtraction concepts are introduced early in math education and negative numbers later. Algebraic reasoning requires students to reason with equivalence and negative signs together.

Many students experience difficulties solving linear algebraic equations that vary in complexity with respect to negative number and equivalence (Trezise & Reeve, under review). Trezise and Reeve manipulated the properties of the equal and negative signs to examine how it affects algebraic problem solving abilities. They found students showed high accuracy for canonical (e.g., $7x + 3 = 24$) and non-canonical (e.g., $51 = 9x + 6$) equations with positive values, but accuracy was much lower for equations with negative number and relational problems. Their findings indicate that students struggled with negative numbers and sophisticated equivalence problem solving. However, it is unknown whether these problem solving patterns accurately represent all students, or whether patterns of algebraic problem solving differ between students. For example, some students may understand negative numbers but not equivalence concepts, and some may have a better understanding of equivalence concepts. It is possible all students acquire both concepts in a similar manner.

1.4. Current research

The current research examined whether algebraic problem difficulty and problem solving time pressure affects math anxiety. Specifically, we (1) use latent variable mixture modelling to classify different patterns of relations between algebraic problem solving and math anxiety, (2) examine whether different math anxiety patterns are differentially related to math problem solving patterns and time pressure, and (3) whether observed patterns are age related.

It should be noted we use the term “worry” rather than anxiety in our research. Worry is the cognitive aspect of anxiety (Deffenbacher, 1980; Eysenck & Calvo, 1992) and has been found to be consistently associated with anxiety-related working memory deficits, and impairments in performance (Eysenck & Calvo, 1992; Morris & Liebert, 1969, 1970; Sari, Koster, & Derakshan, 2017; Trezise & Reeve, 2014a).

[1] Trezise and Reeve's (2014a, 2016) work refers to state working memory, rather than working memory ability per se. Given anxiety is thought to reduce working memory capacity, they measured decreases in working memory capacity when math anxious students encounter mathematics.