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Intraday volatility and network topological properties in the Korean stock market

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ABSTRACT

We examine whether the relationship between market volatility and network properties in the low-frequency level can be applied to the high-frequency level. For the analysis, we use the minimum spanning tree (MST) method constructed from intraday Korean stock market data. The results show that the higher the market volatility is, the denser the MST of stocks becomes. The normalized tree length shows a strong negative relationship with market volatility, indicating that the distances between nodes are shorter when the market volatility is high. The mean occupation layer shows the tendency of having a smaller value in a higher volatility market. The maximum number of links becomes larger when the market volatility increases. All these network properties support the network being dense and shrinking in high market volatility conditions; that is, the degree of co-movement in financial market is reinforced in the intraday high-frequency level.

1. Introduction

The topological perspective on financial markets has received much attention in the field of econophysics [1–9]. A financial market itself is generally considered as a complex system, tangled intricately. Mantegna [4] proposed the transformation of correlation coefficients to the Euclidean distance. The correlation coefficient is calculated from log return of two assets. A lot of research has attempted to apply the minimum spanning tree (MST) method to various financial areas: stock markets (global [10,11], developed market [1,2,9,12–17], emerging market [3,18,19]), interest rates [20,21], currencies [22–24], bonds [25], and commodities [26]. Most of this research is static analysis, such as overviews on the network topology or taxonomic studies in terms of sector, region, or other characteristics. Recently, Coelho et al. [11] attempted a dynamic analysis. Onnela et al. [5] provided the basic idea that the market condition is reflected in the topology of networks in a financial market by comparing the topological difference between Black Monday and a day of normal conditions. Gilmore et al. [25] studied the dynamic co-movement of government bonds in 1998–2006 using hierarchical tree and network properties. Jang et al. [22] showed that the topology of a currency network is changed in a currency crisis. However, these studies did not cover intraday level analysis. On the other hand, the importance of high-frequency intraday analysis of the financial market has been gradually highlighted as the investment time horizon gets shorter thanks to the advances of computing and communication technologies. Accordingly, the interest in volatility is moving from daily toward intraday level [27–29].

Our study focuses on the intraday high-frequency level analysis in the topology of a financial network. Assets are actively traded in a short time horizon and the number of assets traded during the short time period is sufficient to construct a

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network structure. Considering these conditions, we analyze a stock market in which trading is executed in an automated electronic system. By this analysis, we try to extend the previous research to intraday level analysis and to find a more generalized relationship between the volatility of the financial market and the corresponding network properties.

This paper is organized as follows. The following section gives a brief description of the data and provides some preliminary analysis including intraday stylized facts. Section 3 reports our findings on the network properties for various market volatility levels. Finally, a summary and conclusions are presented.

2. Methodology

In this section, we describe the methodology used for the analysis of the data. In order to construct the MST following the method suggested by Mantegna [4], the correlation coefficient should be calculated in the first step. For the preliminary work, the stock price return time series should be calculated from the time series of price. The logarithmic return of stock i over period τ at time t is given by

$$R_i(t,\tau) = \ln P_i(t) - \ln P_i(t-\tau), \tag{1}$$

where $P_i(t)$ is the price of stock i at time t. The correlation coefficient between stock i and stock j at time t over the n time intervals is defined as

$$\rho_{ij}^t = \frac{T\left(R_i^t \cdot R_j^t\right) - \left(R_i^t \cdot \mathbf{1}_T\right)\left(R_j^t \cdot \mathbf{1}_T\right)}{\sqrt{\left(T\left(R_i^t \cdot R_i^t\right) - \left(R_i^t \cdot \mathbf{1}_T\right)^2\right)\left(T\left(R_j^t \cdot R_j^t\right) - \left(R_j^t \cdot \mathbf{1}_T\right)^2\right)}}, \tag{2}$$

where

$$R_i^t = \left(R_i(t-n\tau+\tau), \ldots, R_i(t)\right), \quad i = 1, \ldots, N,$$
$$\mathbf{1}_T = (1, \ldots, 1), \quad T \text{ number of 1s},$$

where T indicates the length of time window: $T = n\tau$. $X \cdot Y$ denotes the inner product of two vectors X and Y. Through the calculation of the correlation coefficients between all pairs of N stocks, the correlation matrix C^t at time t is constructed. C^t is an $N \times N$ symmetric matrix with diagonal elements equal to unity. In the second step, we use the method proposed by Mantegna [4] to construct the MST. All the elements of the correlation matrix C^t are transformed to distances. The distance between stock i and stock j at time t is defined as

$$d_{ij}^t = \sqrt{2(1-\rho_{ij}^t)}. \tag{3}$$

The MST for the N stocks can be obtained through Kruskal's algorithm from the $N \times N$ distance matrix [30]. For the purpose of analyzing the MST, we employ some network properties which display the characteristics of the network. The normalized tree length (NTL), which measures the closeness among the components of network, at time t, is given by

$$NTL_t = \frac{1}{N-1} \sum_{d_{ij}^t \in \Theta} d_{ij}^t, \tag{4}$$

where Θ is the set of edges and d_{ij}^t is the distance between stock i and stock j at time t in Eq. (3).

There are various methods to determine the central node of a network: the highest vertex degree (number of edges), the highest correlation coefficient-weighted vertex degree, and the betweenness centrality [7,22,25]. We adopt the betweenness centrality. The betweenness centrality is defined as

$$B(\upsilon) = \sum_{i \neq \upsilon} \sum_{j \neq \upsilon, j \neq i} \frac{\pi_{ij}(\upsilon)}{\pi_{ij}}, \tag{5}$$

where π_{ij} indicates the number of shortest geodesic paths from node i to node j, and $\pi_{ij}(\upsilon)$ denotes the number of shortest geodesic paths from node i to node j through a node υ.

The level of node i, $\phi(\upsilon_i)$, counts the number of edges connected to the central node through the shortest path from node i. By definition, the level of the central node is zero: $\phi(\upsilon_c) = 0$.

Based on the centrality and the level of the node, the mean occupation layer (MOL) at time t is defined by

$$MOL_t = \frac{1}{N} \sum_{1}^{N} \phi(\upsilon_i). \tag{6}$$

The mean occupation layer is an index for the topological structure of a network. If the value of the MOL is small, the network appears to be dense and its nodes tend to crowd the area around the central node. The maximum number of links, $k_{\max}$, is obtained by counting the number of edges of the most connected node in the network. We define the market volatility of the time window T at time t, σ_t, as the average standard deviation of N stocks. In this paper, we choose the values of $\tau = 1$ and $T = 30$ min.