International Journal of Heat and Mass Transfer 122 (2018) 660–680

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

On the non-optimality of tree structures for heat conduction

Suna Yan [a,*,1], Fengwen Wang [b], Ole Sigmund [b]

[a] School of Mechanical Engineering, State Key Laboratory for Manufacturing Systems Engineering, Xi'an Jiaotong University, 710049 Xi'an, China
[b] Department of Mechanical Engineering, Solid Mechanics, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark

ARTICLE INFO

Article history:
Received 22 November 2017
Received in revised form 18 January 2018
Accepted 26 January 2018

Keywords:
Heat conduction
Volume-to-point problem
Tree structures
Lamellar needle structures
Topology optimization

ABSTRACT

This paper revisits topology optimization of heat conduction structures for minimum thermal compliance and minimum maximum temperature, respectively. For both optimization problems, volume-to-line and volume-to-point structures are optimized based on three material interpolation models describing different design spaces regarding the relation between material density and effective conductivity. The numerical results are backed up by analytical studies. Comparisons of results show that lamellar needle structures, rather than commonly seen tree structures, constitute the optimal topologies for heat conduction. This contradicts the usual hypothesis drawn from the observation of natural transferring systems and designs from numerous related studies. The conclusion still holds when a minimum length scale is imposed for both high and low conductive phases. Finally, the minimum thermal compliance problem and the min-max temperature problem are compared in terms of optimal microstructures. Lamellar microstructures with the normal to the material layers bisecting the gradients of direct and adjoint solutions are optimal for both types of problems. The variable thickness sheet model is optimal only for the self-adjoint minimum thermal compliance problem.

1. Introduction

The volume-to-point problem in heat conduction has attracted considerable attention. A general definition of the problem based on the concept proposed by Bejan [1] is as follows: Consider a finite-size volume in which heat is generated at every point. The volume is cooled through a small patch (heat sink) located on its boundary where the temperature is prescribed. Every point of the volume is made of either high or low conductive material and the volume fraction of high conductive material is finite. Design the distribution of materials such that the heat conduction performance of the volume is maximized. In the extensive literature on this problem, different parameters are used as the metric for the heat conduction performance and different design approaches are proposed, as summarized in Table 1.

At the same time as proposing the volume-to-point problem, Bejan developed the constructal theory [1] to solve it by which material distributions minimizing the maximum temperature were theoretically determined. The smallest building blocks (elemental volumes) were optimized at first in terms of the geometric aspect ratio. To cool a greater volume, elemental volumes were lined up by a new high conductive path and the first assembly was built. Higher order assemblies were assembled successively from lower order ones in a similar way. Although geometric parameters of constructal networks were optimized step by step, no explanation was given on the optimality of the assembling approach which inevitably leads to trees with branches. Later, more exact constructal theory-based derivations were developed in [4] by dropping the assumption on the effective conductivity of the first assembly used in [1]. A new series of constructal trees were obtained, but still left the question on the optimality of the assembling approach unresolved. In [3], Bejan concluded that increasing the complexity of constructal trees makes sense for reducing the maximum temperature of the volume. However, Ghodoossi [5,6] proved that the sequence of constructal designs from simple toward complex trees does not improve the heat conduction performance in general. Marck et al. [7] found that the performance of construtal trees strongly depends on the volume fraction of the high conductive material and not on the structural complexity.

Different from the constructal theory, topology optimization methods provide more degrees of freedom in design and allow to study different objective functions and complex design domains [26]. As shown in Table 1, thermal compliance, maximum temperature and weighted sums of them are the most widely used objective functions. Design approaches employing the SIMP model

* Corresponding author.
E-mail address: sunayan.me@gmail.com (S. Yan).
[1] This work was carried out while Suna Yan was a visiting student at Department of Mechanical Engineering, Technical University of Denmark.

https://doi.org/10.1016/j.ijheatmasstransfer.2018.01.114
0017-9310/

Table 1
Literature on the volume-to-point problem.

Objective function (min)	Approach	Rule	Paper
T_{max}	Constructal theory	Theoretical analysis	[1–7]
C_{th}	TO (Density-based models)	MMA	[8]
T_{max}, T_{av}, C_{th}	TO (SIMP)	MMA	[9]
Weighted sum of T_{max}, T_{av}	TO (SIMP)	MMA	[10]
C_{th}	TO (SIMP)	MMA	[11,12]
C_{th}	TO (SIMP)	Optimality criterion	[13]
Weighted sum of C_{th}, E_{in}	TO (Level set)		[14]
C_{th}	TO (Microstructures)	MMA	[15]
C_{th}	TO (SSM and SIMP)	MMA	[16]
Geometric T_{av}	TO	MMA	[17]
Heat flow path	TO	Evolutionary algorithm	[18]
Entropy generation	TO	Temperature gradient-based principle	[19]
T_{max}, T_{av}	TO	Genetic algorithm	[20,21]
Temperature gradient difference	TO	Cellular automaton	[22]
T_{av}	Generative approach	Growth algorithm	[23]
	Generative approach	Space colonization algorithm and genetic algorithm	[24]
T_{max}	Generative approach	Genetic algorithm	[25]

T_{max}: maximum temperature, T_{av}: average temperature, C_{th}: thermal compliance, E_{in}: interface energy, TO: topology optimization, SIMP: Solid Isotropic Material with Penalization model, SSM: Stiffness Spreading Method.

[27,28], level-set and microstructures have been studied. Gradient-based algorithms and genetic algorithms are used to solve the optimization problems. To explore the effect of design space, Manuel and Lin [8] used 6 fixed and 6 tunable density-based material interpolation models, including the Solid Isotropic Material with Penalization (SIMP) model and the Rational Approximation of Material Properties (RAMP) model [29], for minimizing the thermal compliance. The design results from a uniform initial guess include gray material distributions and discrete trees. In traditional level-set based topology optimization approaches, holes cannot be added to the design domain and thus initial guesses have a crucial effect on design results. To circumvent this problem, Yamada et al. [14] incorporated a fictitious interface energy term into the thermal compliance and reformulated the level set function. The obtained designs for the volume-to-point problem are trees with different degrees of geometry complexity controlled by the weight of the fictitious interface energy term. Li et al. [16] studied the optimization of volume-to-point structures for high and low conduction regions with the addition of high conductive pipes, also resulting in tree-like structures. Changing implementation details of the optimization process leads to different design results despite starting from the same initial guess, which exemplifies the existence of local minima. In [18], Li et al. proposed a concept named heat flow path across which no heat is transferred and thus the heat flow paths are tangent to temperature gradients at every point of the design domain. During the optimization, the distribution of heat flow paths are homogenized by converting elements at low heat flux regions to be low conductive. Different tree-like design results are obtained from different initial guesses.

In the references listed in Table 1 employing topology optimization approaches, all the resulting discrete optimized designs are tree structures with only two exceptions. Based on single precision graphical processing unit (GPU) programming, Wadbro and Berggren [13] optimized a volume-to-point structure using up to one million elements. In addition, a strict convergence condition and a slow continuation approach to update the penalization factor were used. Apparently, in this case the optimized designs became lamellar or needle-like instead of tree structures, even if based on a uniform initial guess. By construction, due to an anisotropic formulation not un-like the microstructure approach used in our work, Dede [15] also obtained lamellar structures instead of trees, but without a density dependence.

Probably inspired by the functional similarity between heat transfer structures and natural mass transfer systems and the branching configurations possessed by the latter, some generative approaches have been developed based on the assumption that dendritic or tree-like topologies are preferable for heat conduction. A volume-to-point problem similar to that in [16] was studied in [23], where high conductive pipes were added into the low conductive base material step-by-step and positions and sizes of high conductive pipes were optimized. Aiming at obtaining branching configurations of high conductive pipes which were conceived as optimal for heat conduction, a branching promotion strategy was developed. The resulting designs are inevitably tree structures. In [24], the distribution of high conductive material was parameterized as the distribution of auxins and extracted by a space colonization algorithm which was chosen specifically to produce complex dendritic structures. By updating the auxin distribution, diversified dendritic topologies were obtained. In [25], the lengths, thicknesses and bifurcation angles of dendritic structures were treated as design variables and optimized by a genetic algorithm to minimize the maximum temperature of the volume. The comparison of obtained trees at different bifurcation levels showed that complexity, i.e. volume-filling, is a common feature of good designs.

Starting from Murray's law [30], and further inspired by all the above works, researchers have gradually realized that branching and tree-like topologies are a common feature of natural volume-to-point systems. An overwhelming majority of the designs from the intensive study on the volume-to-point problem are hence tree-like, independent of whether the design approaches are targeted at tree structures or not. Remarks like "it can be seen that the design results resemble natural tree networks" can be found in related literature [1,9,12,19,22,24], implying that tree structures have been accepted as the optimal topologies of volume-to-point structures. *However, the study in this paper shows that lamellar needle-like structures instead of tree structures are the true optimal topologies for volume-to-point structures in the context of steady-state heat conduction for minimum thermal compliance and minimum maximum temperature, respectively. Hence transferring optimization cannot be the pure reason for natural transfer systems being trees.* Other physics, load fluctuations, structural requirements, or the need for damage adaption [31,32] may play an important role in inducing the morphogenesis of branches and even loops.

The intuitive explanation for tree structures not being optimal is simple: The most efficient way to transport heat from a point in the structure to the sink is a straight line. Hence, any detour, as provided by tree structures, is less efficient. Establishing