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On some nonlocal equations with competing coefficients

Tingxi Hu [a], Lu Lu [b,*]

[a] Department of Mathematics, Central China Normal University, Wuhan 430079, PR China
[b] School of Statistics and Mathematics, Zhongnan University of Economics and Law, Wuhan 430079, PR China

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ABSTRACT

In this paper we study the following nonlocal problems

$$\begin{cases} -\left(a+b\displaystyle\int_{\mathbb{R}^3}|\nabla u|^2\right)\Delta u+V(x)u=Q(x)u^p, & x\in\mathbb{R}^3,\\ u\in H^1(\mathbb{R}^3), \quad u>0, \quad x\in\mathbb{R}^3, \end{cases}$$

where the constants $a,b>0$, $3<p<5$, $V(x)$ and $Q(x)$ are two functions in $C^{\gamma}_{loc}(\mathbb{R}^3)\bigcap L^{\infty}(\mathbb{R}^3)$. By comparing the decay rate of $V(x)$ and $Q(x)$, we first obtain two theorems stating the existence of positive ground states. Under certain assumptions on $Q(x)$, we further prove the existence of positive bound states by using a linking argument with a barycenter map restricted on a Nehari manifold.

1. Introduction

In this paper we investigate the existence of positive solutions of the following nonlocal problem with subcritical nonlinearity:

$$\begin{cases} -\left(a+b\displaystyle\int_{\mathbb{R}^3}|\nabla u|^2\right)\Delta u+V(x)u=Q(x)|u|^{p-1}u, & x\in\mathbb{R}^3,\\ u\in H^1(\mathbb{R}^3), \quad u>0, \quad x\in\mathbb{R}^3, \end{cases} \tag{P}$$

where $a,b>0$ are constants, $3<p<5$, $V(x)$ and $Q(x)$ are two positive functions such that $\lim_{|x|\to\infty}V(x)=V_\infty>0$, $\lim_{|x|\to\infty}Q(x)=Q_\infty>0$.

* Corresponding author.
E-mail addresses: tingxihu@mails.ccnu.edu.cn (T. Hu), lulu@zuel.edu.cn (L. Lu).

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Problem (P) is related to the stationary analogue of equation

$$\rho\frac{\partial^2 u}{\partial t^2}-\left(\frac{P_0}{h}+\frac{E}{2L}\int_0^L\left|\frac{\partial u}{\partial x}\right|^2dx\right)\frac{\partial^2 u}{\partial x^2}=0$$

proposed by Kirchhoff in [19] as an extension of classical D'Alembert's wave equation for vibration of elastic strings. Kirchhoff's model takes into account the changes in length of the string produced by transverse vibrations, so the nonlocal term appears. We refer [5,26] for physical and numerical aspects of Kirchhoff's model.

In the past few years, the following nonlocal problem

$$\begin{cases}-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\right)\Delta u+V(x)u=f(x,u), & x\in\mathbb{R}^3,\\ u>0,\ u\in H^1(\mathbb{R}^3),\end{cases}\tag{1.1}$$

has been studied extensively, where $V:\mathbb{R}^3\to\mathbb{R}$, $f\in C(\mathbb{R}^3\times\mathbb{R},\mathbb{R})$ and $a,b>0$ are constants. Various results on the existence of positive solutions, multiple solutions, sign-changing solutions, ground states and semiclassical states have been obtained, see for examples [12,15,16,23,27,30] and the references therein. However, few work concern the effective of the asymptotic behavior at infinity of the potentials. In the present paper, we make some contributions in this direction.

Set $V(x)=V_\infty+\lambda h(x)$, $Q(x)=Q_\infty+q(x)$ in problem (P), where $\lambda\geq 0$ is a parameter. Then problem (P) becomes

$$\begin{cases}-\Big(a+b\int_{\mathbb{R}^3}|\nabla u|^2\Big)\Delta u+V_\infty u+\lambda h(x)u=\Big(Q_\infty+q(x)\Big)|u|^{p-1}u, & x\in\mathbb{R}^3,\\ u\in H^1(\mathbb{R}^3),\quad u>0,\quad x\in\mathbb{R}^3.\end{cases}\tag{P_λ}$$

We assume that $h(x)$ and $q(x)$ belong to $C^{\gamma}_{loc}(\mathbb{R}^3)\cap L^\infty(\mathbb{R}^3)$ for some $0<\gamma<1$ such that

(H1) $h(x)\gneqq 0$, $\lim_{|x|\to\infty}h(x)=0$;
(H2) $\kappa:=Q_\infty+\inf\limits_{x\in\mathbb{R}^3}q(x)>0$, $q(x)\not\equiv 0$, $\lim_{|x|\to\infty}q(x)=0$.

If $q(x)\geq 0$, we consider the existence of ground state of (P_λ) by comparing the decay rate of $h(x)$ and $q(x)$. On the other hand, if $q(x)\leq 0$ and $|q(x)|$ is not large, the existence of bound states of (P_λ) is also established.

Let $H^1(\mathbb{R}^3)$ be the Sobolev space equipped with the inner product and norm

$$(u,v)=\int_{\mathbb{R}^3}\Big(a\nabla u\nabla v+V_\infty uv\Big)dx\quad\text{and}\quad\|u\|=(u,u)^{1/2}.$$

We shall seek solutions of (P_λ) as critical points of the energy functional $I_\lambda:H^1(\mathbb{R}^3)\to\mathbb{R}$ associated with (P_λ) and given by

$$I_\lambda(u)=\frac{\|u\|^2}{2}+\frac{\lambda}{2}\int_{\mathbb{R}^3}h(x)|u|^2dx+\frac{b}{4}\left(\int_{\mathbb{R}^3}|\nabla u|^2dx\right)^2-\frac{1}{p+1}\int_{\mathbb{R}^3}\Big(Q_\infty+q(x)\Big)|u|^{p+1}dx.\tag{1.2}$$

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