



On an inverse spectral problem for first-order integro-differential operators with discontinuities



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ABSTRACT

A convolution integro-differential operator of the first order with a finite number of discontinuities is considered. Properties of its spectrum are studied and a uniqueness theorem is proven for the inverse problem of recovering the convolution kernel along with the boundary condition from the spectrum.

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1. Introduction

The paper deals with an inverse spectral problem for the integro-differential operator that corresponds to the following boundary value problem $L = L(H(x), h, \{a_k\}_{k=1}^N, \{\alpha_k\}_{k=1}^N)$ with discontinuities inside the interval:

$$\ell y(x) := iy'(x) + \int_0^x H(x-t)y(t) dt = \lambda y(x), \quad x \in \bigcup_{j=0}^N (a_j, a_{j+1}), \quad (1)$$

$$y(a_j + 0) = \alpha_j y(a_j - 0), \quad j = \overline{1, N}, \quad (2)$$

$$U(y) := hy(0) - y(\pi) = 0, \quad (3)$$

where $H(x)$ is a complex-valued function, $(\pi - x)H(x) \in L_2(0, \pi)$, $h, \alpha_1, \dots, \alpha_N \in \mathbb{C} \setminus \{0\}$, $0 = a_0 < a_1 < a_2 < \dots < a_N < a_{N+1} = \pi$.

Inverse problems of spectral analysis consist in recovering operators from their spectral characteristics. Such problems often appear in mathematics, mechanics, physics, electronics, geophysics, meteorology and

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other branches of natural sciences and engineering. For differential operators with discontinuities inverse problems were studied in [1–11]. The presence of a discontinuity in the mathematical model is connected with discontinuous material properties or with a discontinuity in the corresponding physical process.

For integro-differential and other classes of non-local operators inverse problems are more difficult for investigation, and the classical methods either are not applicable to them or require essential modifications. Various aspects of inverse problems for integro-differential operators without discontinuities were studied in [12–24] and other papers. The presence of the discontinuity conditions (2) essentially complicates studying even the direct problem for L , i.e. the problem, when its spectral properties are to be established. In this paper a special fundamental solution of Eq. (1) is constructed, which allowed to study the properties of the spectrum of L . Namely, the following theorem holds.

Theorem 1. *The problem L has infinitely many eigenvalues λ_k , $k \in \mathbb{Z}$, of the form*

$$\lambda_k = 2k + \omega + \kappa_k, \quad \{\kappa_k\} \in l_2, \quad (4)$$

with account of multiplicities. Moreover, $\omega = (i\pi)^{-1}(\ln(\alpha_1 \dots \alpha_N) - \ln h)$.

Consider the following inverse problem.

Inverse Problem 1. Given the spectrum $\{\lambda_k\}_{k \in \mathbb{Z}}$, find the function $H(x)$ and the number h , provided that the values a_j , α_j , $j = \overline{1, N}$, are known a priori.

We will study this inverse problem in the case, when the pair of sets $\{a_k\}_{k=1}^N$ and $\{\alpha_k\}_{k=1}^N$ satisfies the following condition (*Condition \mathcal{A}*): $a * b(x) := \int_0^x a(t)b(x-t) dt \neq 0$ for $x \in (0, \pi]$, where $a(x) = A_k := \alpha_0 \alpha_1 \dots \alpha_k$, $x \in (a_k, a_{k+1})$, $k = \overline{0, N}$, $\alpha_0 = 1$, and $b(x) = A_N / A_{N-k}$, $x \in (\pi - a_{N+1-k}, \pi - a_{N-k})$, $k = \overline{0, N}$. The case, when Condition \mathcal{A} is not fulfilled, requires a separate investigation. Obviously, Condition \mathcal{A} is automatically fulfilled, if all $\alpha_k > 0$. Moreover, it is easy to prove that in the case $N = 1$ Condition \mathcal{A} is equivalent to the condition $\alpha_1 \notin (-\infty, 0]$. In [25] for the case, when $N = 1$ and $a_1 = \pi/2$, the following theorem was proven, which gives uniqueness of solution along with necessary and sufficient conditions for the solvability of **Inverse Problem 1** in this special case.

Theorem 2 (See [25]). *Let $N = 1$, $a_1 = \pi/2$ and $\alpha_1 \notin (-\infty, 0]$. Then for an arbitrary sequence of complex numbers $\{\lambda_k\}_{k \in \mathbb{Z}}$ of the form (4) there exists a unique (up to values on a set of measure zero) function $H(x)$, $(\pi - x)H(x) \in L_2(0, \pi)$, and a unique number $h \neq 0$, such that $\{\lambda_k\}_{k \in \mathbb{Z}}$ is the spectrum of the corresponding boundary value problem $L(H(x), h, \pi/2, \alpha_1)$.*

The proof of **Theorem 2** in [25] is based on the method used in [15], which gives also an algorithm for solving the inverse problem. We note that the general case of $\{a_k\}_{k=1}^N$ is more difficult, even in studying the direct spectral problem. Here we restrict ourselves only to the following uniqueness theorem and use another approach than in [25].

Theorem 3. *Let Condition \mathcal{A} be fulfilled. Then the specification of the spectrum $\{\lambda_k\}_{k \in \mathbb{Z}}$ uniquely determines the function $H(x)$ along with the number h .*

We note that uniqueness of the solution of an inverse problem for a second-order integro-differential operator with a single discontinuity of a special type was discussed in [26].

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