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Relative risk aversion and wealth dynamics

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Abstract

As a follow-up to the work of Chen and Huang [S.-H. Chen, Y.-C. Huang, Risk preference, forecasting accuracy and survival dynamics: simulations based on a multi-asset agent-based artificial stock market, Working Paper Series 2004-1, AI-ECON Research Center, National Chengchi University, 2004; S.-H. Chen, Y.-C. Huang, Risk preference and survival dynamics, in: T. Terano, H. Kita, T. Kaneda, K. Arai, H. Deghchi (Eds.), Agent-Based Simulation: From Modeling Methodologies to Real-World Applications, Springer Series on Agent-Based Social Systems, vol. 1, 2005, pp. 135–143], this paper continues to explore the relationship between wealth share dynamics and risk preferences in the context of an agent-based multi-asset artificial stock market. We simulate a multi-asset agent-based artificial stock market composed of heterogeneous agents with different degrees of relative risk aversion. As before, we find that the difference in risk aversion and the resultant saving behavior are the primary forces in determining the survivability of agents. In addition to the stability of the saving behavior, the level of the saving rate also plays a crucial role. The agents with stable saving behavior, e.g., the log-utility agents, may still become extinct because of their low saving rates, whereas the agents with unstable saving behavior may survive because of their high saving rates, implied by their highly risk-averse preferences.

Keywords: Risk preferences; CRRA (constant relative risk aversion); Blume–Easley theorem; Agent-based artificial stock markets; Genetic algorithms

1. Motivation and introduction

The contribution of *risk preference* to the survivability (wealth share) of investors has recently received a series of theoretic and simulation studies (e.g., [1,18,17,2,4,5]). The results are mixed, depending on how we approach this issue. While the standard analytic approach proves the irrelevance of risk preference to survivability [17,2], the agent-based computational approach indicates the opposite [4,5]. This kind of inconsistency, as quite often seen in the agent-based computational economics literature, simply reflects the sensitivity of the classical (analytical) results to the interacting heterogeneous boundedly-rational behavior.

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Ref. [4] actually supports an earlier result obtained in [1], which is also known as the *Kelly criterion* in financial economics. This result basically points out the optimal type of risk preference, namely, the *CRRA* (*constant relative risk aversion coefficient*) *agent with an RRA coefficient of one*. Equivalently, it is the log utility function. [5] reestablishes this result, while in an agent-based computational setting. They examine the long-run wealth share dynamics of eleven different types of CRRA agents, with RRA coefficients ranging from 0 to 1 with increments of 0.1. They find that in finite time the wealth share is positively related to the CRRA coefficient, and in the long run, only the agents with high CRRA coefficients can survive. All others become extinct.

This paper is an extension of [5] in the sense that we wish to extend the earlier testing domain of the CRRA coefficient from $[0,1]$ to an even larger positive domain. In doing so, we are inquiring whether a higher degree of risk aversion can actually enhance the survivability of agents. Notice that the degree of risk aversion is not the original concern of either the Kelly criterion or the Blume–Easley theorem [1], both of which are only concerned with the dominance of the log-utility type agents. Risk aversion is involved because the log-utility agent is also known as a CRRA type of agent with an RRA coefficient of one. Now, is this the optimal degree of risk aversion? Will more risk-averse agents (i.e., those with RRA coefficients greater than one) be also driven out of the market when they are competing with the log-utility agents? Or, would higher risk aversion help them survive? These are the questions that we try to answer in this paper.

We consider these questions to be particularly relevant because the empirical literature actually suggests a large range of relative risk aversion coefficients. Some of them are exactly one or less than one, but many more are greater than one. Of course, it is doubtful whether one can directly compare our results with those empirical values, since they refer to quite different stories. However, given the prevailing empirical results on high risk aversion, it is definitely useful to know what makes them so, and our agent-based computational setting can serve as a good starting point.

The remainder of this paper is organized as follows. Section 2 gives a brief introduction to a simple multi-asset model, which is originally used in [1] and later extended and modified by [17]. Section 3 presents the artificial multi-asset market, which is an agent-based version of the analytical model presented in Section 2. Section 4 gives the experimental design. To justify the range of the relative risk aversion coefficient considered in this paper, it starts with a brief review of the literature on the empirical estimation of the RRA coefficient in Section 4.1, followed by the setting of other control parameters in Section 4.2. The simulation results are presented and discussed in Section 5, and are followed by the concluding remarks in Section 6.

2. A simple multi-asset model

The simulations presented in this paper are based on an agent-based version of the multi-asset market as [1,17] have studied. The market is complete in the sense that the number of states is equal to the number of assets, say M. At each date t, the outstanding volume of each asset is exogenously fixed at one unit. There are I investors in the market, each indexed by i. At time t asset m will pay dividends w_m if the corresponding state m occurs, and 0 otherwise. The behavior of states follows a finite-state stochastic process, which does not have to be stationary. The dividends w_m will be distributed among the I investors proportionately according to their owned share of the respective asset. The dividends can only be either re-invested or consumed. Hoarding is prohibited. If agent i chooses to consume c, her satisfaction is measured by her utility function $u(c)$. This simple multi-asset market clearly defines an optimization problem for each individual.

$$\max_{\{\{\delta^i_{t+r}\}_{r=0}^{\infty},\{\alpha^i_{t+r}\}_{r=0}^{\infty}\}} \quad E\left\{\sum_{r=0}^{\infty}(\beta^i)^r u^i(c^i_{t+r})|B^i_t\right\} \tag{1}$$

$$\text{subject to} \quad c^i_{t+r}+\sum_{m=1}^{M}\alpha^{i,*}_{m,t+r}\cdot\delta^{i,*}_{t+r}\cdot W^i_{t+r-1}\leqslant W^i_{t+r-1},\quad \forall r\geqslant 0, \tag{2}$$

$$\sum_{m=1}^{M}\alpha^i_{m,t+r}=1,\quad \alpha^i_{m,t+r}\geqslant 0,\ \forall r\geqslant 0. \tag{3}$$

In Eq. (1), u^i is agent i's temporal utility function, and β^i, also called the discount factor, reveals agent i's time preference. The expectation $E()$ is taken with respect to the most recent belief B^i_t, which is a probabilistic