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Local and consistent centrality measures in parameterized networks[☆]

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HIGHLIGHTS

- Axiomatic approach to characterize centrality measures.
- Only recursive centrality measures.
- Use consistency axiom.
- Our axiomatic characterization highlights the conceptual similarities among recursive centrality measures.

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ABSTRACT

We propose an axiomatic approach to characterize centrality measures for which the centrality of an agent is recursively related to the centralities of the agents she is connected to. This includes the Katz–Bonacich and the eigenvector centrality. The core of our argument hinges on the power of the consistency axiom, which relates the properties of the measure for a given network to its properties for a reduced problem. In our case, the reduced problem only keeps track of local and parsimonious information. Our axiomatic characterization highlights the conceptual similarities among those measures.

1. Introduction

Centrality is a fundamental concept in network analysis. Bavelas (1948) and Leavitt (1951) were among the first to use centrality to explain differential performance of communication networks and network members on a host of variables including time to problem solution, number of errors, perception of leadership, efficiency, and job satisfaction.

Following their work, many researchers have investigated the importance of the centrality of agents on different outcomes. Indeed, it has been shown that centrality is important in explaining employment opportunities (Granovetter, 1974), exchange networks (Cook et al., 1983; Marsden, 1982), peer effects in education and crime (Calvó-Armengol et al., 2009; Haynie, 2001; Hahn et al., 2015), power in organizations (Brass, 1984), the adoption of innovation (Coleman et al., 1966), the creativity of workers (Perry-Smith and Shalley, 2003), the diffusion of microfinance programs (Banerjee et al., 2013), the flow of information (Borgatti, 2005; Stephenson and Zelen, 1989), the formation and performance of R&D collaborating firms and inter-organizational networks (Boje and Whetten, 1981; Powell et al., 1996; Uzzi, 1997), the success of open-source projects (Grewal et al., 2006) as well as workers' performance (Mehra et al., 2001).

While many measures of centrality have been proposed,[1] the category itself is not well defined beyond general descriptors such as node prominence or structural importance. There is a class of centrality measures, called prestige measures of centrality, where the centralities or statuses of positions are recursively related to the centralities or statuses of the positions to which they are connected. Being chosen by a popular individual should add more to one's popularity. Being nominated as powerful by someone seen by others as powerful should contribute more to one's perceived

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[1] See Wasserman and Faust (1994) and Jackson (2008) for an introduction and survey.

power. Having power over someone who in turn has power over others makes one more powerful. This is the type of centrality measure that will be the focus of this paper.

It includes the degree centrality, the Katz–Bonacich centrality (due to Katz, 1953, and Bonacich, 1987) and the eigenvector centrality. Take, for example, the Katz–Bonacich centrality of a particular node. It counts the total number of paths that start from this node in the graph, weighted by a decay factor based on path length. This means that the paths are weighted inversely by their length so that long, highly indirect paths count for little, while short, direct paths count for a great deal. Another way of interpreting this path-based measure is in terms of an intuitive notion that a person's centrality should be a function of the centrality of the people he or she is associated with. In other words, rather than measure the extent to which a given actor "knows everybody", we should measure the extent to which the actor "knows everybody who is anybody".

While there is a very large literature in mathematical sociology on centrality measures (see e.g. Borgatti and Everett, 2006; Bonacich and Lloyd, 2001; Wasserman and Faust, 1994), little is known about the foundation of centrality measures from a behavioral viewpoint.[2] Ballester et al. (2006) were the first to provide a microfoundation for the Katz–Bonacich centrality. They show that, if the utility of each agent is linear–quadratic, then, under some condition, the unique Nash equilibrium in pure strategies of a game where n agents embedded in a network simultaneously choose their effort level is such that the equilibrium effort is equal to the Katz–Bonacich centrality of each agent. This result is true for any possible connected network of n agents. In other words, Nash is Katz–Bonacich and the position of each agent in a network fully explains her behavior in terms of effort level.

In the present paper, we investigate further the importance of centrality measures in economics by adopting an *axiomatic* approach. We derive characterization results not only for the Katz–Bonacich centrality but also for two other centrality measures, namely the degree centrality and the eigenvector centrality, which all have the properties that one's centrality can be deduced from one's set of neighbors and their centralities.

Our characterization results are based on three key ingredients, namely the definitions of a *parameterized* network, of a *reduced parameterized* network and the *consistency* property.

A *parameterized network* is defined as a set of vertices and edges for which some of the vertices, that we call *terminal* vertices, are assigned a positive real number. Conceptually, one can interpret a parameterized network as a set of regular vertices and their neighbors such that the centrality of those neighbors, the terminal vertices, has been parameterized and no longer needs to be determined. In the context of social networks, parameterized networks correspond to what Banerjee et al. (unpublished) study in Indian rural communities: the centrality of prominent individuals in villages (here the terminal nodes) is public knowledge and can be considered as a parameter.

A *reduced parameterized network* is defined from an initial parameterized network together with a vector of centralities. It is a small world that consists in a subset of regular vertices of the initial parameterized network and their neighbors. The terminal vertices in the reduced network are assigned a positive number, which is either taken from the initial network or from the vector of centralities.

These two definitions are instrumental in order to characterize centrality measures when combined with the *consistency* property. This property requires that the centralities in the initial network are also the centralities in the reduced networks constructed from the initial network and its vector of centralities.

As stressed by Aumann (1987), consistency is a standard property in cooperative game as well as noncooperative game theory. It has been used to characterize the Nash equilibrium correspondence (Peleg and Tijs, 1996), the Nash bargaining solution (Lensberg, 1988), the core (Peleg, 1985) and the Shapley value (Hart and Mas-Colell, 1989; Maschler and Owen, 1989), to name a few. As nicely exposed by Thomson (2011), consistency expresses the following idea. A measure is consistent if, for any network in the domain and the "solution", it proposes, for this network, the "solution" for the reduced network obtained by envisioning the departure of a subset of regular vertices with their component of the solution is precisely the restriction of the initial solution to the subset of remaining regular vertices. Consistency can be seen as a robustness principle, it requires that the measure gives coherent attributes to vertices as the network varies.

The usefulness of the consistency property for characterization purposes depends on how a reduced problem is defined. In our case, it is very powerful since a reduced problem only keeps track of local and parsimonious information, namely the set of neighbors and the centrality of those neighbors.

Contrary to the Nash equilibrium approach (Ballester et al., 2006), we believe that our *axiomatic* approach allows us to understand the relationship between different centrality measures, i.e. the degree, the Katz–Bonacich and the eigenvector centrality measure. This is important because as stated above, different types of centralities can explain different behaviors and outcomes. For example, the eigenvector centrality seems to be important in the diffusion of a microfinance program in India (Banerjee et al., 2013). On the contrary, the Katz–Bonacich centrality seems to be crucial in explaining educational and crime outcomes (Haynie, 2001; Calvó-Armengol et al., 2009) and, more generally, outcomes for which complementarity in efforts matter. The degree centrality is also important. For example, Christakis and Fowler (2010) combine Facebook data with observations of a flu contagion, showing that individuals with more friends were significantly more likely to be infected at an earlier time than less connected individuals.

The axiomatic approach is a standard approach in the cooperative games and social choice literature but axiomatic characterizations of centrality measures are scarce. Boldi and Vigna (2014) propose a set of three axioms, namely size, density and score monotonicity axioms, and check whether they are satisfied by eleven standard centrality measures but do not provide characterization results. Garg (2009) characterizes some centrality measures based on shortest paths. Kitti (2016) provides a characterization of eigenvector centrality without using consistency. There is also a literature in economics and computer science that provides axiomatic foundations for ranking systems, see e.g. Altman and Tennenholtz (2008), Demange (2014), Henriet (1985), Rubinstein (1980) and van den Brink and Gilles (2000). This literature does not use the consistency property.

The closest paper to ours is Palacios-Huerta and Volij (2004), who have used an axiomatic approach, and in particular a version of the consistency property, to measure the intellectual influence based on data on citations between scholarly publications. They find that the properties of invariance to reference intensity, weak homogeneity, weak consistency, and invariance to splitting of journals characterize a unique ranking method for the journals. Interestingly, this method, which they call the invariant method (Pinski and Narin, 1976) is also at the core of the methodology used by Google to rank web sites (Page et al., 1998). The main difference with our approach is the way Palacios-Huerta and Volij (2004) define a reduced problem. In their paper, a reduced problem is non-parameterized in the sense that it only contains vertices and edges. As a consequence, they need to impose an ad hoc formula to split

2 For surveys of the literature on networks in economics, see Jackson (2008, 2014), Ioannides (2012), Jackson and Zenou (2015) and Jackson et al. (2017).